Binding Energies

The Binding Energy of any nucleus ${}^A_Z X$ is the energy required to separate the nucleus into free neutrons and protons. It can be determined using atomic masses, $M({}^1H)$ and $M({}^A_Z X)$:

$$B({}^A_Z X) = \left[N m_n + Z M({}^1H) - M({}^A_Z X)\right] c^2$$

It is the binding energy that ultimately is an important determinator of nuclear stability. If B is positive, the nuclide is said to be stable against dissociating into free neutrons and protons. It is necessary to generalize this because a nucleus containing A nucleons is said to be stable if its mass is smaller than that of any other possible combination of A nucleons. The binding energy of a nucleus ${}^A_Z X$ against dissociation into any other possible combination of nucleons, for example nuclei R and S is:

$$B = \left[M(R) + M(S) - M({}^A_Z X)\right] c^2$$

The separation energy is the energy required to remove one proton (or neutron) from a nuclide. Even, however, with negative B values, there may be additional reasons for a stable nucleus.

Example: show that ${}^8Be$ has a positive binding energy but is unstable with respect to decay into two alpha particles.

The binding energy is determined by:

$$B({}^A_Z X) = \left[4 m_n + 4M({}^1H) - M({}^A_Z X)\right] c^2$$

$$= \left[4(1.008665u) + 4(1.007825u) - 8.005305u\right] c^2 \left(\tfrac{931.5MeV}{c^2 u}\right) = 56.5Mev$$

Now to calculate the binding energy with respect to decay into 2 alpha particles, we have:

$$B = \left[2M(\alpha) - M({}^8Be)\right] c^2 = \left[2(4.002602)u - 8.005305u\right] c^2 \left(\tfrac{931.5MeV}{c^2 u}\right)$$

$$= -0.093MeV$$

${}^8Be$ is unstable against decay into 2 alpha particles. From purely an energy standpoint, there should be no reason to expect that this disintegration won't happen. Experimentally ${}^8Be$ does decay into 2 alpha particles. This is partially responsible for the preponderance of He in stars ... the helium can't easily combine into a stable nuclei to form heavier nuclei.

Here is a reference off the internet:
http://www.nucleonica.net

In particular you might want to see the decay scheme java applet.

In your text you have a plot of the known nuclides which shows neutron number vs proton number. For A<40, stable nuclei are formed when $Z \sim N$

The explanation behind this is due to electrostatic repulsion ... in a sense the neutrons serve to separate protons thus reducing the electrostatic repulsion. However, with too many neutrons, the strong nuclear force is less effective which tends to limit the benefits of additional neutron addition. For A>40, additional neutrons are required to obtain stable nuclei.

In the 1930's a model of the nucleus developed which is essentially a liquid drop model. They were able to develop what is called the semi-empirical mass formula which is:

$$B\left({}^{A}_{Z}X\right) = a_V A - a_A A^{2/3} - \frac{3}{5}\frac{Z(z-1)e^2}{4\pi\varepsilon_0 r} - a_S \frac{(N-Z)^2}{A} + \delta$$

a_V is a volume term that indicates the binding energy is about the sum of all the interactions between the nucleons.

The nuclear force is short range enough so that it really only interacts with nearest neighbors.

The second term is the surface effect is thought of somewhat in the same idea as surface tension .. the nucleons on the surface are not completely surrounded by other nucleons. This is essentially a correction to the volume term, reducing the overall interaction.

The third term represents the coulomb interaction, but not from the nucleon with itself.

The fourth term is a symmetry term which is quantum mechanical in origin.

The fifth term is a pairing energy and reflects the observation that the nucleus is more stable for even-even nuclides.

Some of the values for the various constants are:

$$a_V = 14\text{MeV}$$

$$a_A = 13\text{MeV}$$

$$a_S = 19\text{MeV}$$

$$\text{pairing}: \delta = \begin{cases} +\Delta : \text{even}-\text{even} \\ 0 : \text{odd}-\text{even} \\ -\Delta : \text{odd}-\text{odd} \end{cases} \quad : \Delta = 33\text{Mev} \times A^{-3/4}$$

This model has been pretty good at explaining the observed binding energy of the nuclides. It does describe quite a bit but keep in mind that it has 7 parameters … if you can't fit a curve with 7 parameters, you might as well give up.

The radius appearing in the Coulomb energy is given by an earlier result:

$$R = r_0 A^{1/3}$$

Radioactive Decay 12.6

The general form of radioactivity is the same for all decays because of the statistical nature of the decay process. The activity is defined as the number of disintegrations per unit time. If you have N unstable atoms of a material, the activity is given by:

$$\text{Activity} = -\frac{dN}{dt} = R$$

the – sign is inserted to insure that R is positive. The Si unit of activity is the becquerel which is 1 decay/s. A more common unit is the curie (Ci) which is $3.7\text{x}10^{10}$ decays/s. Apparently federal licensing for radioactive materials is at levels above 10^4Ba.

if N(t) is the number of radioactive nuclei in a sample at time t, and λ (the decay constant) is the probability per unit time that any given nucleus will decay, then the activity is given by:

$$R = \lambda N(t)$$

The number dN of nuclei decaying during the time interval dt is:

$$dN(t) = -Rdt = -\lambda N(t)dt$$

We can write this as:

$$\frac{dN}{N} = -\lambda dt \Rightarrow \ln(N) = -\lambda t + \text{constant}$$

if $N(t=0)=N_0$ then we obtain the radioactive decay law:

$$N(t) = N_0 e^{-\lambda t}$$

This radioactive decay is consistent with observations.
We can also write this in terms of the activity:

$$R = R_0 e^{-\lambda t}$$

You have probably heard the term half-life. This is a more common reference when talking about nuclear decay and it is the time it takes for ½ of the radioactive nuclei to decay:

$$N(t_{1/2}) = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \Rightarrow \ln\left(\frac{1}{2}\right) = \ln\left(e^{-\lambda t_{1/2}}\right) = -\lambda t_{1/2} \Rightarrow t_{1/2} = \frac{\ln(2)}{\lambda} = \frac{0.693}{\lambda}$$

The average lifetime is given by:

$$\tau = \frac{1}{\lambda}$$

You have, no doubt, heard of carbon 14 dating. Let's see how this works.
The decay scheme for the production of radioactive carbon is this:

$$n + {}^{14}N \rightarrow {}^{14}C + p$$

and this happens in the upper atmosphere. A natural ratio of ${}^{14}C$ to ${}^{12}C$ exists in molecules of CO_2 which living organisms take in. When living organisms die, they quit taking in ${}^{14}C$ and the ratio between the two carbons decreases with time. There is evidence that indicates that just before 9000 years ago, the ratio of ${}^{14}C$ to ${}^{12}C$ was about 1.5 times higher than it is today. However, using evidence from tree rings, the counting of decay rates has been calibrated back to about 10,000 years. The half-life of ${}^{14}C$ is 5730 years, and thus it is reasonable to date objects up to about 45,000 years.

Here is an example:

A bone suspected to have originated during the period of the roman emperors was found in Great Britain. The ratio ${}^{14}C$ to ${}^{12}C$ of is found to be 1.1×10^{-12}. How old is the bone? The initial ratio of ${}^{14}C$ to ${}^{12}C$ at the time of death is about $R_0=1.2\times10^{-12}$. The number of ${}^{14}C$ atoms decays as the exponential:

$$N\left({}^{14}C\right) = N_0 e^{-\lambda t}$$

The ratio of ions is given by:

$$R = \frac{N\left({}^{14}C\right)}{N\left({}^{12}C\right)} = \frac{N_0\left({}^{14}C\right)e^{-\lambda t}}{N\left({}^{12}C\right)} = R_0 e^{-\lambda t}$$

$$\Rightarrow e^{-\lambda t} = \frac{R}{R_0} \Rightarrow t = \frac{-\ln(R/R_0)}{\lambda} = -t_{1/2}\frac{\ln(R/R_0)}{\ln(2)} = -5730\frac{-0.087}{0.693} = 719\text{yr}$$

The bone is not of Roman origin.