Magnetic Effects: The normal Zeeman effect

We have till now the quantum numbers n, L, m_l with their restrictions:

$$n > 0$$

$$\ell < n$$

$$|m_\ell| \leq \ell$$

The Angular momenta are given by:

$$L = \sqrt{\ell(\ell+1)}\hbar : L_z = m_\ell \hbar$$

This gives us a degeneracy in the energies, which it turns out can be partially lifted by the application of an external magnetic field.

Let's look at the magnetic effects …

think of the electron in orbit as if it were a classical current. The magnetic moment of the current is defined by:

$$\mu = IA$$

(we did this in general and fundamental physics also).

Working with this expression we have (for T=1 period of oscillation):

$$\mu = \tfrac{q}{T} A = \frac{(-e)}{\left[\frac{2\pi r}{v}\right]}\left[\pi r^2\right] = \tfrac{-erv}{2} = -\tfrac{e}{2m}(rp) = -\tfrac{e}{2m} L$$

This then gives the connection between the magnetic moment and the angular momentum:

$$\vec{\mu} = -\tfrac{e}{2m}\vec{L}$$

of course, this is perfectly classical at this point in actuality. However, be aware that for the magnetic moment of the electron in orbit, the direction is opposite to that of the angular momentum.

If a magnetic dipole is placed in an external magnetic field, the dipole will experience a torque:

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

and this torque tends to align the dipole with the magnetic field. We have also seen in general physics that the potential energy of a magnetic dipole in an external magnetic field is given by:

$$V_B = -\vec{\mu} \bullet \vec{B}$$

To remind you of how this occurred, the work done by a torque is given by:

$$W = \int_{\theta_1}^{\theta_2} \mu B \sin(\theta) d\theta = -\mu B \cos(\theta)\Big|_{90}^{\theta} = -\mu B \cos(\theta) = -\vec{\mu} \bullet \vec{B}$$

There is a great similarity between the action of the magnetic field on the dipole and a spinning top … the magnetic field will result in a precession of the dipole about the magnetic field. In addition, the presence of the magnetic field establishes a preferred direction in space, namely along the z-axis. We can't have perfect alignment along this direction since the application of B results in spatial quantization. In any event, the magnetic field along this direction will be given by:

$$\mu_z = \tfrac{e\hbar}{2m} m_\ell = -\mu_B m_\ell$$

We have defined a new quantity called the Bohr Magneton:

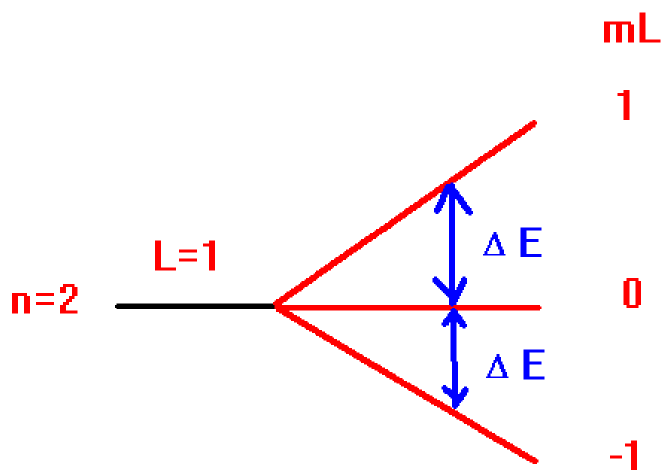
$$\mu_B \equiv \frac{e\hbar}{2m}$$

What about the energy of the orbiting electron in an external magnetic field? What we find is that the potential energy is now split into 2l+1 different energies since

$$V_B = -\mu_z B = +\mu_B m_\ell B$$

We say that the degeneracy has been lifted by the magnetic field.
It would be nice if the story ended there but it does not.

Here is how the levels will split in hydrogen:



See Figure 7.6 Page 253 for more information here.

Essentially when you look at transitions from the 2p state of hydrogen to the 1s state (l=0) you see now 3 distinct lines (transitions) where as previously only 1 line was observed.

However, the “selection rule” for mL will not allow more than three different lines in the normal Zeeman effect.

Now starting in the 1920’s efforts were begun to detect the effects of space quantization by measuring energy differences in an external magnetic field. The ultimate experiment that proved successful was that of Stern and Gerlach in 1922. In this experiment, an oven was used to produce a beam of p-state atoms. These atoms were passed through an inhomogenous magnetic field so that different orientations of the magnetic moment would experience different forces. The particular force on a magnetic atom is given by:

$$F_z = -\frac{dV_B}{dz} = \mu_z \frac{dB}{dz}$$

Note about this force: this is closely related to the connection which we have seen previously for a force and a conservative potential:

$$\vec{F} = -\vec{\nabla} V$$

We also used this in 2nd semester physics to show the connection between the electric field and the electric potential. It's the same idea here.

Clearly then there will only be a force on the dipoles if there is an inhomogenous magnetic field in their presence. It may be said that a field gradient needs to exist.

If this gradient does exist, there will be three different outcomes, depending upon the value of m_L for the p-state atoms. Stern and Gerlach did their experiment with silver atoms and observed 2 distinct lines not three. Clearly space was quantized but they really did scratch their heads about the absence of the third line. Here is a link to a java applet related to this classical experiment.

http://www.physik.rwth-aachen.de/~harm/aixphysik/atom/SternGerlach/.

Notice that instead of observing 3 lines, two lines are observed!

7.5 Intrinsic Spin

The clear evidence for spatial quantization from several similar experiments could not be refuted. However, the absence of the third line is the kind of thing that would have made most people not publish their data. It happens this way … the experiment was repeated, and on different systems also with similar results. To Stern and Gerlach's credit, they went ahead and published although they clearly could not completely explain their results. It was clear that there was a problem.

Wolfgang Pauli was the first to suggest the presence of yet a fourth quantum number assigned to the electron which might account for anomalous optical spectra. His reasoning was that there were 4 coordinates … 3 space and 1 time so there ought to be 4 quantum numbers. He might have even thought about the possibility that time was quantized, but then again perhaps not. If I had been him, I would have considered it at the time.

In 1925 Goudsmit and Uhlenbeck (two graduate students in Holland) proposed that the electron must have an intrinsic angular momentum and therefore a magnetic moment (since the electron is charged). Ehrenfest though showed that if the electron were thought of as a cloud then at the surface of the electron, a point would need to be moving faster than the speed of light. I suppose that the moral to this story is be careful what you propose. How was this resolved? The answer was that if this were true, then such an intrinsic angular momentum could only be a purely quantum mechanical effect, having no classical analogue. Note: they made no mention of the experiment of Stern and Gerlach although this very experiment provided evidence for their proposal. Perhaps they were bothered by the "p" state.

Goudsmit and Uhlenbeck proposed that the electron must have an intrinsic spin quantum number S=1/2 and that the spinning electron reacts similarly to an orbiting electron in a magnetic field. If the analogy to orbital angular momentum held, then there would be

2S+1=2(1/2)+1=2 components of the spin angular momentum. Thus, the proposal was that the magnetic spin quantum number has two values: “up” and “down” or $m_s = \pm\frac{1}{2}$, and the electron can never be spinning with its magnetic moment μ_S directly pointed along the z-axis. This means that for each quantum state previously identified by: $[n, \ell, m_\ell]$ that two additional states must exist so that the system is now identified by: $[n, \ell, m_\ell, s, m_s]$. These states would be degenerate unless the atom is in an external magnetic field. In a magnetic field the energies will separate and you would say that the degeneracy has been lifted.

The intrinsic spin angular momentum vector S has a magnitude of $|\vec{S}| = \sqrt{s(s+1)}\hbar = \sqrt{\frac{3}{4}}\hbar$

The magnetic moment is $\vec{\mu}_e = -\frac{e}{m}\vec{S}$, or $\vec{\mu}_e = -2\mu_B \frac{\vec{S}}{\hbar}$ where the factor of 2 comes about as a consequence of relativity (the Dirac Equation) which we’ll not work with. In terms of the gyromagnetic ratios, then:

$$\vec{\mu}_\ell = -\frac{\mu_B \vec{L}}{\hbar} = -\frac{g_\ell \mu_B \vec{L}}{\hbar}$$

$$\vec{\mu}_s = -2\frac{\mu_B \vec{S}}{\hbar} = -\frac{g_s \mu_B \vec{S}}{\hbar}$$

The z-component of S is $S_z = m_s\hbar = \pm\frac{\hbar}{2}$

Again you can calculate the angle of inclination of the spin from:

$$\cos(\theta) = \frac{S_z}{S} = \pm\frac{1}{2\sqrt{\frac{3}{4}}} = \pm\frac{1}{\sqrt{3}} \Rightarrow \theta = \pm 54.7^0$$

It is now clear why the Stern and Gerlach experiment produced only two distinct lines: the lines they were observing were due to the electron spin and these would be split even without an external magnetic field since classically the electron sees the proton spinning around it thus producing an external magnetic field which the intrinsic magnetic moment of the electron must interact with.
http://plato.stanford.edu/entries/physics-experiment/app5.html
less obvious from this discussion is the gem gathered from the link above: in fact the atomic beam that Stern and Gerlach used was not a p state but instead an s state. This means that the separation into 3 components would never have happened anyway. Operations on an s state will only be responding to the intrinsic spin of the electron.

Energy Levels and Electron Probabilities
http://www.physics.northwestern.edu/ugrad/vpl/atomic/hydrogen.html

The link above will sketch the hydrogen atom energy levels for you. Each level is characterized by 4 unique quantum numbers. It does it for both the Bohr atom and the Schrödinger Solution.

Till now, we have not said anything about selection rules. Firstly, since photons carry an angular momentum of 1 the change in angular momentum for an electron undergoing a transition is then also one. Thus: for “allowed” transitions

$$\Delta \ell = \pm 1$$

Other transitions may also occur, for instance two photon absorption.
In accord with this we have then:

$$\Delta m_\ell = \pm 1, 0$$

And no restriction is placed upon n (except for energy conservation):

$$\Delta n = \text{anything}$$

These transitions are most probable to occur.
Look at example 7.10 on page 261. (new edition)