De Broglie Waves

(1982 – 1987)

Awareness of Einstein's work showing mass-energy equivalence. 1924 Thesis: at the Faculty of Sciences at Paris University *Recherches sur la Théorie des Quanta* Basically he postulated that matter must have wave-like properties:

$$\lambda = \frac{h}{p}$$

This may be how he arrived at this hypothesis: For a photon:

$$E = pc$$

which was known classically. The energy was shown to be given by:

$$E = hf$$

Thus if the two expressions are equated, we find:

$$hf = pc \Rightarrow h = p\frac{c}{f} = p\lambda \Rightarrow \lambda = \frac{h}{p}$$

De Broglie's extension was to suggest that this simple relationship held for all particles, not just photons. This is now called the De Broglie wavelength of a particle.

example: What is the De Broglie wavelength of a tennis ball of mass 70 g traveling at 25 m/s and an electron of energy 50 eV.

For the tennis ball:

$$\lambda = \frac{h}{p} = \frac{6.63 \mathrm{x} 10^{-34}\,\mathrm{Js}}{0.07\,\mathrm{kg} \,\mathrm{x}\, 25\,\mathrm{m/s}} = 3.8 \,\mathrm{x}\, 10^{-34}\,\mathrm{m}$$

For the electron, first recognize that it is produced by accelerating an electron through a potential difference of 50 V. For the non-relativistic electron we then have:

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE} = \frac{\sqrt{2(mc^2)E}}{c} \Rightarrow \lambda = \frac{hc}{\sqrt{2(mc^2)E}} = \frac{1240\,\mathrm{eV\,nm}}{\sqrt{2 \,\mathrm{x}\, 0.511 \,\mathrm{x}\, 10^6\,\mathrm{eV} \,\mathrm{x}\, 50\,\mathrm{eV}}}$$

$$\Rightarrow \lambda = 0.17\,\mathrm{nm}$$

This is a very important result since it says that acceleration of electrons through only a moderate potential difference will produce a wavelength on the order of crystalline structure size. As we will see, this accidental correlation becomes significant.

Let's look at Bohr's quantization condition in the light of the De Broglie wavelength.

Suppose we have standing waves on the Bohr orbitals. In order for a standing wave to exist, we have:

$$n\lambda = 2\pi r$$

where r is the orbital radius. We now use the De Broglie relation here:

$$n\left(\frac{h}{p}\right) = 2\pi r$$

The angular momentum of the electron is given by L=rp. We thus have:

$$n\left(\frac{h}{2\pi}\right) = rp = L \Rightarrow L = n\hbar$$

This means that Bohr's assumption follows as a very natural consequence of the condition for a standing wave to exist from De Broglie's wavelength. This is wonderful, but your author says that verification of the wave nature of electrons is still lacking.

A rather simple experiment of Davisson Germer (1927) at Bell Labs turns out to have provided verification of the wave nature of electrons.

I'll leave it up to you to read the text here regarding the background. Suffice it to say that not every accident is bad.

In any event, the pair observed sharp peaks at an angle of about 50 degrees in the scattered electrons from their experiment. This was only observed after they had cleaned their crystal (accomplished by heating it up to remove oxygen). According to http://www.keytometals.com/Article32.htm:

> **Annealing.** A heat treatment designed to produce a recrystallized grain structure and softening in work-hardened alloys. Annealing usually requires temperatures between 705 and 1205^{o}C, depending on alloy composition and degree of work hardening.

So if some surface regions of the nickel that they were working with went through these temperatures, then they were able to produce single crystal domains in their nickel. Ultimately, this was correctly interpreted as evidence for the wave nature of the electron.
One of the quotes that I have for 1kg of Nickel Bullion is about $15 (2014); I think we can only guess that it was more expensive in 1927.
In lab, we will also scatter electrons from a target and directly observe this wave nature associated with the electron.

Time to relocate to our lab for the demonstration.