Physics 240: Unquiz 01

A ball is thrown upward from an initial position of

$$\vec{R} = 0\,\hat{x} + 9\,\hat{y}\,\mathrm{m}$$

with an initial velocity vector given by

$$\vec{v} = 0\,\hat{x} + 5\,\hat{y}\,\frac{\mathrm{m}}{\mathrm{s}} \quad .$$

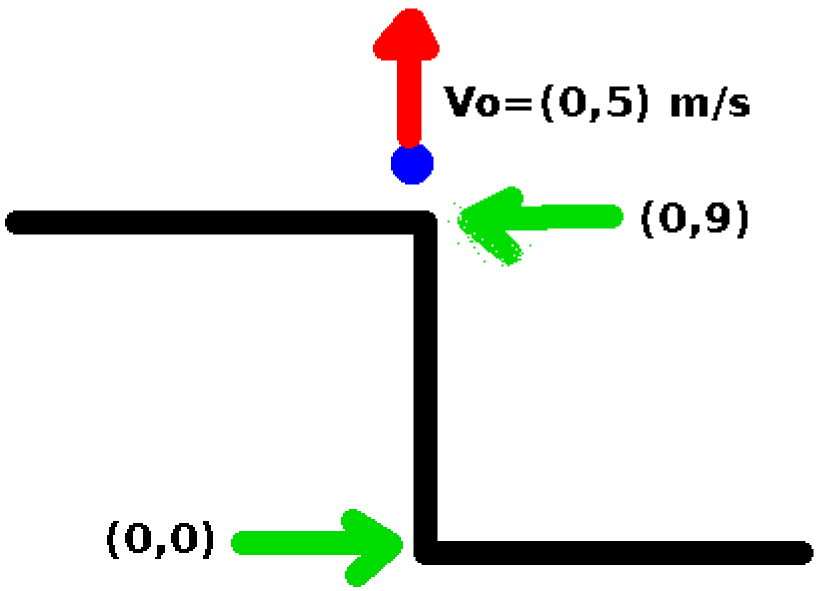
(1) How high does the ball travel above the coordinate given by:

$$\vec{P} = 0\,\hat{x} + 0\,\hat{y} \quad ?$$

(2) Find the velocity vector when the ball strikes the ground.

(3) How long is the ball in flight?

Physics 240: Unquiz 01 Solution



A ball is thrown upward from an initial position of $\vec{R}=0\,\hat{x}+9\,\hat{y}\,m$ **with an initial velocity vector given by** $\vec{v}=0\,\hat{x}+5\,\hat{y}\,\frac{m}{s}$ **.**

(1) How high does the ball travel above the coordinate given by:

$$\vec{P}=0\,\hat{x}+0\,\hat{y}\ \ \textbf{?}$$

Since there is no initial velocity in the x direction, this problem reduces to a 1-dimensional problem. Near the Earth, the acceleration due to gravity is given by $\vec{a}=-g\,\hat{y}$ where in SI units $g=9.8\,\frac{m}{s^2}$. The three equations of motion are:

$$y=y_0+v_{y,0}t+\frac{1}{2}a_y t^2 : v_y=v_{y,0}+a_y t : v_y^2=v_{y,0}^2+2\,a_y\,\Delta y$$

here, replace a_y with -g to have:

$$y=y_0+v_{y,0}t-\frac{1}{2}g\,t^2 : v_y=v_{y,0}-g_y\,t : v_y^2=v_{y,0}^2-2\,g\,\Delta y$$

The question can now be answered most directly by noting that at the highest point, the y component of the velocity is zero. We can then use the 3rd equation which will give the maximum height above the point from which it was thrown:

$$v_y^2=v_{y,0}^2-2\,g\,\Delta y \Rightarrow \Delta y=\frac{v_y^2-v_{y,0}^2}{-2g}$$

The initial conditions are: $v_y=0 : v_{y,0}=9$ Now find Δy:

$$\Delta y=\frac{0-5^2}{-2\,x\,9.8}=\frac{25}{19.6}=1.28\,m$$

This is the maximum height above the point from which the ball was thrown. To find the maximum height above the ground, it is necessary to add the initial height to this which gives:

$$height=9.00+1.28=10.28\,m$$

(2) Find the velocity vector when the ball strikes the ground.

There are several ways to answer this now. Perhaps the easiest is to use the last equation, with the initial conditions, to find the final velocity. When the ball hits the ground, it is at a position given by P above. From this we can calculate $V_y^2 = V_{y,0}^2 - 2g(\Delta y) \Rightarrow v_y = \pm\sqrt{2(9.8)(10.28)} \Rightarrow v_y = -14.2\,m/s$

Since the final velocity is downward, the physical answer is -14.2 m/s. Thus the final velocity vector is:

$$\vec{v}_f = 0\,\hat{x} - 14.2\,\hat{y}\,\frac{m}{s}$$

(3) How long is the ball in flight?

It is possible to use either the first equation or the second equation to answer this. The second equation is easier:

$$v_y = v_{y,0} + a_y t \Rightarrow -14.2 = 5 - 9.8\,t \Rightarrow t = 1.96\,s$$

Using the first equation should give the same result.

$$y = y_0 + v_{y,0} t + \frac{1}{2} a_y t^2 \Rightarrow 0 = 9 + 5t - 4.9t^2 \Rightarrow -4.9t^2 + 5t + 9 = 0 \Rightarrow t = \frac{-5 \pm \sqrt{25 + 176.5}}{-2 \times 4.9}$$

$$t = 1.96\,s \text{ or } t = -0.96\,s$$

The correct answer here is the one with positive time (t=1.96 s)

Here, in the context of the problem and by the statement of the problem, the physical answer is 1.96 s. The other solution answers a different question: how much before t=0 would the ball have to be thrown up so that at t=0, the velocity was (0,5) and the position was (0,9). (We know that the velocity at that point was +14.2 m/s by symmetry).