Total mechanical energy:

1/2 of the time average energy is kinetic, 1/2 is potential

$$\langle E\rangle = \langle K\rangle + \langle U\rangle$$

For a spring mass system:

$$x(t) = A\cos(\omega t+\phi) : v(t) = -\omega A\sin(\omega t+\phi) ; \omega = \sqrt{\frac{k}{m}}$$

If the spring is at a maximum displacement, the total energy is $E = \frac{1}{2}kA^2$.

Calculate the time average kinetic energy and the time average potential energy:

$$\langle K\rangle = \frac{1}{2}m\langle v^2\rangle = \frac{1}{2}m\omega^2A^2\langle \sin^2(\omega t+\phi)\rangle = \frac{1}{2}m\omega^2A^2\left(\frac{1}{2}\right)$$

$$m\omega^2 = m\left(\sqrt{\frac{k}{m}}\right)^2 = m\frac{k}{m} = k \Rightarrow \langle K\rangle = \frac{1}{2}kA^2\left(\frac{1}{2}\right)$$

$$\langle U\rangle = \frac{1}{2}k\langle x^2\rangle = \frac{1}{2}kA^2\langle \cos^2(\omega t+\phi)\rangle = \frac{1}{2}kA^2\left(\frac{1}{2}\right)$$

$$\langle K\rangle + \langle U\rangle = \frac{1}{2}kA^2\left(\frac{1}{2}\right) + \frac{1}{2}kA^2\left(\frac{1}{2}\right) = \frac{1}{2}kA^2 = \langle E\rangle = \frac{1}{2}kA^2$$

Note: to calculate the time average of cos or sin:

$$1 = \cos^2(\theta) + \sin^2(\theta) : \langle 1\rangle = \langle \cos^2(\theta)\rangle + \langle \sin^2(\theta)\rangle$$

cosine and sine behave exactly the same except displaced by a phase. This means each contributes equally to the value of 1. Thus:

$$\langle \cos^2(\theta)\rangle = \langle \sin^2(\theta)\rangle = \frac{1}{2}$$