	Physics 240: Worksheet 29	Name: ________________

(1) Suppose, as the result of a thunderclap that the air surrounding the thunderhead was initially at a temperature of 27 ^{0}C and a pressure of 1 Atm. and then adiabatically increased to a pressure of 1.25 atm. What was the change in temperature of this region of air? Assume that for air, $\gamma=1.4$

(2) Suppose you have 2 moles of an ideal monatomic gas in 3 dimensions at an initial pressure of 1 atm and an initial temperature of 300K.
(a) What is the volume of the IDG?

(b) Fill in the following table.

Process	**Q**	**W**	**ΔU**
T->2T Isovolumeric (V=constant)			
T->2T Isobaric (P=constant)			
V->2V Isothermal (T=constant) (at 300K)			
T->2T Adibatic (Q=0)			

(3) Suppose that 2 moles of an IDG go through the following process:
(1) T_i=300K, T_f=600K adiabatic process.
(2) $V_i=0.05\text{m}^3$, $V_f=0.10\text{m}^3$ isothermal process
(3) T_i=600K, T_f=300K, adiabatic process
(4) $V_i=0.10\text{m}^3$, $V_f=0.05\text{m}^3$ isothermal process
Calculate the following quantities:
(a) The total heat added to the process
(b) The total heat evolved by the process
(c) The net heat added to (or evolved from) the system.
(d) The Work done by the system along the entire process.
(e) ΔU for this entire closed process.
(f) Define the engine's efficiency by: $\varepsilon \equiv \frac{W}{Q_{\text{input}}}$ Find out how efficient this engine is.

(4) Show for the closed process involving 2 isotherms and 2 adiabatic processes as in problem (3) that the efficiency is given by: $\varepsilon_c = 1 - \frac{T_{\text{cold}}}{T_{\text{hot}}}$

(5) What is the efficiency of a Carnot cycle which is operating between the temperature extremes of 0^{0}C and -200 ^{0}C.

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Thermodynamic processes with **the ideal gas** (this applies only to the IDG!)

One of the most important things to keep in mind throughout this discussion is the first law of thermodynamics:

$$\Delta U = Q - W$$

In words, this means that the change in internal energy is equal to the net heat **added** to the system minus the work done **by the system**.

Note again: physics sign convention for work

+W means work was done **by the system**: if a gas expands (and moves a piston against a pressure) the system does work. This work is positive in our convention.

If at a constant pressure, then $W = P\Delta V = |P\Delta V|$

-W means work was done **on the system**: if a gas compresses (and moves a piston against a pressure) work is done on the system. This work is negative in our convention.

If at a constant pressure, then $W = P\Delta V = -|P\Delta V|$

Let's investigate some particular processes now.

We have already defined the work done by the system (so long as it is a thermodynamic system involving the variables P, V and T) as:

$$W = P(\Delta V).$$

If the pressure and volume change, the calculation gets more complicated:

$$W = \sum_i P_i [\Delta V_i]$$

Heat can be added to the ideal gas in a number of ways. We'll be considering these.

(1) Suppose that heat is added so that $\Delta V=0$ (isovolumeric). Then:

$$W = 0 \text{ and } \Delta U = Q$$

We need to know how the heat is added. This is given by:

$$Q = nc_v(\Delta T)$$

where c_v is called the "***molar specific heat at a constant volume***." Thus, it's easy to calculate the change in internal energy here, provided you know c_v (which, for an ideal gas, we will calculate later):

$$\Delta U = nc_v(\Delta T)$$

To emphasize: no work was done by or on the system. All heat added or removed from the system directly changed the internal energy of the system only.

(2) Suppose that heat is added so that $\Delta P=0$ (isobaric). Then:

$$W = P(\Delta V) \text{ and } Q = nc_p(\Delta T)$$

where c_p is called the "***molar specific heat at a constant pressure***." It is pretty easy to see that

$$c_p > c_v$$

and we can even determine by how much larger it is for the ideal gas (which we are getting ready to calculate). We can thus see that if the gas goes through an isobaric process, then

$$\Delta U = Q - W \Rightarrow \Delta U = nc_p(\Delta T) - P(\Delta V)$$

Now, you will find out from the kinetic theory of an ideal gas that the internal energy of an ideal gas is a function of temperature only (so long as n remains constant, as it does in our class).

Consider the comparison of an isovolumeric process between the same two temperatures as an isobaric process. Both processes must produce the same ΔU since the temperature change for the two processes is the same. Thus:

$$\left[\Delta U_{\text{isovolumeric}} = \Delta U_{\text{isobaric}}\right] \Rightarrow nc_v\left(\Delta T\right) = nc_p\left(\Delta T\right) - P\left(\Delta V\right)$$

We can write ΔV (remember: for the RHS of the equation, P is constant) in terms of the other thermodynamic variables:

$$V = nR\frac{T}{P} \Rightarrow \Delta V = nR\frac{\Delta T}{P}$$

Thus we can further simplify the comparison to read:

$$\left[\Delta U_{\text{isovolumeric}} = \Delta U_{\text{isobaric}}\right] \Rightarrow nc_v\left(\Delta T\right) = nc_p\left(\Delta T\right) - nR\left(\Delta T\right)$$

It is now a very simple step in algebra to show:

For an Ideal Monatomic Ideal Gas

$$c_v = c_p - R \Rightarrow c_p = c_v + R$$

I promised you it needed to be larger and now you see that it is.

(3) We can have another process with an ideal gas, namely an isothermal process where ΔT=0. Note: just because you have an isothermal process, this does not mean that Q=0. Hmmm… temperature does not always have to change if you add Q to a system.

Ok, enough with the foreshadowing … if you have an isothermal process, then ΔU=0 (why? read the bold italic section above).

For such a process, you can't define an appropriate heat capacity because for you to use a heat capacity, temperature has to change. You might wrongly think, then, that Q=0 for this process. It is not … let's see what it is:

Since ΔT=0, we must have ΔU=0. This then means that the first law of thermodynamics looks like this:

$$\Delta U = Q - W = 0 \Rightarrow Q = W$$

Ok, so the way you find Q is by finding out how much work was done. Let's do that (again):

The work done is given by:

$$W = P(\Delta V)$$

Here, however, P is not constant so you would need to add up small increments of work. We can use an approximation I introduced last time. First, write P in a different form:

$$P = nR\frac{T}{V}$$

T is constant here. We use this to find the work:

$$W = nRT\frac{\Delta V}{V}$$, or more precisely, $$W = nRT\sum_i \frac{\Delta V_i}{V_i}$$

From the approximation I showed in the last sheet, you know to write this as a log:

$$W = nRT\ln\left(\frac{V_f}{V_i}\right)$$

Calculus students have a much easier time:

$$W = nRT\int_{V_i}^{V_f}\frac{dV}{V} = nRT\ln\left(\frac{V_f}{V_i}\right)$$

Ok, we can now calculate the heat added in an isothermal process:

$$Q = nRT\ln\left(\frac{V_f}{V_i}\right)$$

(4) An adiabatic process: we can also take the ideal gas through an adiabatic process in which Q=0, and you now know that this is not the same as an isothermal process. Now, this is going to be a bit more difficult than the other processes but I'll provide you with the details so you won't need imagination to know that this is right. We'll get as a consequence the derivation of the "adiabatic equation of state" for an ideal gas. This is extremely important if you want to understand how heat engines work efficiently.

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Derivation of the IDG adiabatic equation of state (non-calculus)

For an adiabatic process, Q=0. This means that we have from the first law:

$$[\Delta U = Q - W] \Rightarrow \Delta U = -W$$

Ok, so far so good. If we calculate W, then we've calculated ΔU for this process. However, that turns out to be easier to say than to do but let's do it anyway. We can't directly calculate W as we did before because P,V and T are all changing here. Never-the-less, we do have to remember that no matter what the process is, if it operates between two temperature extremes, ΔU is given by:

$$\Delta U = nc_v(\Delta T)$$

Thus, we can write:

$$nc_v(\Delta T) = -P(\Delta V) = -nRT\left(\tfrac{\Delta V}{V}\right) \Rightarrow nc_v\left(\tfrac{\Delta T}{T}\right) = -nR\left(\tfrac{\Delta V}{V}\right)$$

Now, we know that R is connected to c_p since $R = c_p - c_v$. So we could write:

$$c_v\left(\tfrac{\Delta T}{T}\right) = -\left(c_p - c_v\right)\left(\tfrac{\Delta V}{V}\right) = \left(c_v - c_p\right)\left(\tfrac{\Delta V}{V}\right)$$

Let's divide by c_v:

$$\left(\tfrac{\Delta T}{T}\right) = \left(1 - \tfrac{c_p}{c_v}\right)\left(\tfrac{\Delta V}{V}\right)$$

Now, apply the useful approximation to both sides of this equation:

$$\ln\left(\tfrac{T_f}{T_i}\right) = (1-\gamma)\ln\left(\tfrac{V_f}{V_i}\right)$$

Gosh, we're almost there! Here,

$$\gamma \equiv \frac{c_p}{c_v}$$

Now you need a useful but often forgotten property of logarithms:

$$a\ln(b) = \ln\left(b^a\right)$$

This means that we can write our relation in a "simpler" form as:

$$\ln\left(\tfrac{T_f}{T_i}\right) = \ln\left(\left[\tfrac{V_f}{V_i}\right]^{1-\gamma}\right)$$

Next, take both sides of this to the power e and use the identity function:

$$e^{\ln(b)} = b$$

This gives us:

$$\tfrac{T_f}{T_i} = \left[\tfrac{V_f}{V_i}\right]^{1-\gamma}$$

Let's lose the "f" subscript on T and V since we want to apply this at any temperature T.

Next, a simple step in algebra puts is in a more familiar form:

$$\tfrac{T}{T_i} = \left[\tfrac{V}{V_i}\right]^{1-\gamma} \Rightarrow T^1 T_i^{-1} = V^{1-\gamma} V_i^{-(1-\gamma)}$$

$$\Rightarrow T^1 V^{\gamma-1} = T_i V_i^{\gamma-1} \Rightarrow T(V)^{\gamma-1} = T_i(V_i)^{\gamma-1}$$

Since T_i and V_i are constants, we thus have the adiabatic equation of state:

$$TV^{\gamma-1} = \text{constant}$$

We can also write this in terms of the pressure (since V=nRT/P) as:

$$\tfrac{PV}{nR}(V)^{\gamma-1} = PV^{\gamma}\left(\tfrac{1}{nR}\right) \Rightarrow PV^{\gamma} = \text{constant}$$

where this constant ***is not the same*** as in the previous expression. Also this constant is a very unusual constant in physics: it has fractional units and the powers of these units may even change as the nature of the gas changes.

Derivation of the IDG adiabatic equation of state (calculus)

For an adiabatic process, Q=0. This means that we have from the first law:

$$[\Delta U = Q - W] \Rightarrow \Delta U = -W$$

Ok, so far so good. If we calculate W, then we've calculated ΔU for this process. However, that turns out to be easier to say than to do but let's do it anyway. We can't directly calculate W as we did before because P,V and T are all changing here. Never-the-less, we do have to remember that no matter what the process is, if it operates between two temperature extremes, dU is given by:

$$dU = nc_v(dT)$$

Thus, we can write:

$$nc_v(dT) = -P(dV) = -nRT\left(\tfrac{dV}{V}\right) \Rightarrow c_v\left(\tfrac{dT}{T}\right) = -nR\left(\tfrac{dV}{V}\right)$$

Now, we know that R is connected to c_p so we could write:

$$c_v\left(\tfrac{dT}{T}\right) = -\left(c_p - c_v\right)\left(\tfrac{dV}{V}\right) = \left(c_v - c_p\right)\left(\tfrac{dV}{V}\right)$$

Let's divide by c_v:

$$\left(\tfrac{dT}{T}\right) = \left(1 - \tfrac{c_p}{c_v}\right)\left(\tfrac{dV}{V}\right) = (1-\gamma)\left(\tfrac{dV}{V}\right);\ \gamma \equiv \tfrac{c_p}{c_v}$$

We can integrate both sides of this differential equation:

$$\int_{T_i}^{T_f}\left(\tfrac{dT}{T}\right) = (1-\gamma)\int_{V_i}^{V_f}\left(\tfrac{dV}{V}\right) \Rightarrow \ln\left(\tfrac{T_f}{T_i}\right) = (1-\gamma)\ln\left(\tfrac{V_f}{V_i}\right)$$

Gosh, we're almost there!

Now you need a useful but often forgotten property of logarithms:

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This means that we can write our relation in a "simpler" form as:

$$\ln\left(\tfrac{T_f}{T_i}\right) = \ln\left(\left[\tfrac{V_f}{V_i}\right]^{1-\gamma}\right)$$

Next, take both sides of this to the power e and use the identity function:

$$e^{\ln(b)} = b$$

This gives us:

$$\tfrac{T_f}{T_i} = \left[\tfrac{V_f}{V_i}\right]^{1-\gamma}$$

Let's lose the "f" subscript on T and V since we want to apply this at any temperature T.

Next, a simple step in algebra puts is in a more familiar form:

$$\tfrac{T}{T_i} = \left[\tfrac{V}{V_i}\right]^{1-\gamma} \Rightarrow T(V)^{\gamma-1} = T_i(V_i)^{\gamma-1}$$

Since T_i and V_i are constants, we thus have the adiabatic equation of state:

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We can also write this in terms of the pressure (since V=nRT/P) as:

$$\tfrac{PV}{nR}(V)^{\gamma-1} = PV^{\gamma}\left(\tfrac{1}{nR}\right) \Rightarrow PV^{\gamma} = \text{constant}$$

where this constant ***is not the same*** as in the previous expression. Also this constant is a very unusual constant in physics: it has fractional units and the powers of these units may even change as the nature of the gas changes.

Let's recap all that you're learned now. In a later lecture I will show you that c_v obeys:

Model	c_v
3-D monatomic gas	$\frac{3}{2}R$
2-D monatomic gas	$\frac{2}{2}R$
1-D monatomic gas	$\frac{1}{2}R$
3-D diatomic solid-gas (lower temperatures)	$\frac{3}{2}R$
3-D diatomic solid-gas (moderate temperatures)	$\frac{3}{2}R+\frac{2}{2}R=\frac{5}{2}R$
3-D diatomic solid-gas (higher temperatures)	$\frac{3}{2}R+\frac{2}{2}R+\frac{2}{2}R=\frac{7}{2}R$

½ R is added to C_v for each degree of freedom which is realized in a system. To understand where these results come from, we'll need to delve into the kinetic theory of an ideal gas. Do, however, remember

$$c_p = c_v + R$$

and

$$\gamma \equiv \frac{c_p}{c_v}$$

Process	**Eqtn of state**	**Q**	**W**	**ΔU=Q-W**
V=constant Isovolumeric	PV=nRT $P \propto T$ $\frac{P}{T} = \text{constant}$	$Q = nc_v(\Delta T)$	0	$\Delta U = nc_v(\Delta T)$
P=constant Isobaric	PV=nRT $V \propto T$ $\frac{V}{T} = \text{constant}$	Q=$nc_p(\Delta T)$	$W = P(\Delta V)$	$\Delta U = nc_v(\Delta T)$
T=constant Isothermal	PV=nRT $P \propto \frac{1}{V}$ $PV = \text{constant}$	Q=W	$W = nRT\ln\left(\frac{V_f}{V_i}\right)$	$\Delta U = 0$ $>> \text{Why?} <<$
Q=0 Adiabatic	PV=nRT $PV^{\gamma} = \text{constant}$ $TV^{\gamma-1} = \text{constant}$	Q=0	$W = -\Delta U$	$\Delta U = nc_v(\Delta T)$

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(1) Suppose, as the result of a thunderclap that the air surrounding the thunderhead was initially at a temperature of 27 ^{0}C and a pressure of 1 Atm. and then adiabatically increased to a pressure of 1.25 atm. What was the change in temperature of this region of air? Assume that for air, γ=1.4

Solution:
We'll need the adiabatic equation of state here, but in a slightly different form:

$$PV^{\gamma} = P\left(nR\tfrac{T}{P}\right)^{\gamma} = \left(nR\right)^{\gamma} P\left(\tfrac{T}{P}\right)^{\gamma} \Rightarrow P^{1-\gamma}T^{\gamma} = \text{constant}$$

For the adiabatic process, we have:

$$P_1^{1-\gamma}T_1^{\gamma} = P_2^{1-\gamma}T_2^{\gamma} \Rightarrow T_2^{\gamma} = T_1^{\gamma}\left(\tfrac{P_1}{P_2}\right)^{1-\gamma}$$

We know what the ratio of pressures is:

$$\frac{P_1}{P_2} = \frac{1}{1.25} = 0.8$$

Now if you use T=27 here, you've fallen into my temperature trap!
You must convert this temperature to K: T=300K
Thus,

$$T_2^{\gamma} = T_1^{\gamma}\left(\tfrac{P_1}{P_2}\right)^{1-\gamma} = \left(300\right)^{1.4}\left(0.8\right)^{-0.4} = \left(2937.4\right)\left(1.0934\right) = 3211.6$$

So,

$$T_2 = \left(3211.6\right)^{\frac{1}{\gamma}} = \left(3211.6\right)^{\frac{1}{1.4}} = 319.7\text{K}$$

Thus,

$$\Delta T = 319.7\text{K} - 300\text{K} = 19.7\text{K}$$

What if the thunderbolt adiabatically doubled the volume?

Here is an important note: The adiabatic process equation and also the ideal gas equations of state must be **simultaneously** obeyed for an adiabatic process. The adiabatic process simply refers to a path.

(2) Suppose you have 2 moles of an ideal monatomic gas in 3 dimensions at an initial pressure of 1 atm and an initial temperature of 300K.
(a) What is the volume of the IDG?

$$V = \frac{nRT}{P} = \frac{2(8.314)300}{1x10^5} = 0.05m^3$$

Fill in the following table.
Here, c_v is 3/2 R because of the description.

Process	**Q**	**W**	**ΔU**
T->2T Isovolumeric (V=constant)	$Q = nc_v(\Delta T)$ $Q = 2(\frac{3}{2})R(600-300)$ $Q = 7483J$	W=0	$\Delta U = nc_v(\Delta T)$ $\Delta U = 7483J$
T->2T Isobaric (P=constant)	$Q = nc_p(\Delta T)$ $Q = n(c_v + R)(\Delta T)$ $Q = 2(\frac{3}{2}R + R)(600-300)$ $Q = 2(\frac{5}{2}R)(300)$ $Q = 12471J$	$W = P(\Delta V)$ $\Delta V = \frac{nR}{P}(\Delta T)$ $P\Delta V = nR(\Delta T)$ $W = nR(\Delta T)$ $W = 2(R)(600-300)$ W=4988.4J	(1) $\Delta U = nc_v(\Delta T)$ $\Delta U = 7483J$ (2) $\Delta U = Q - W$ $\Delta U = 7483J$
V->2V Isothermal (T=constant) (at 300K)	Q=W Q=3458J	$W = nRT\ln(\frac{V_f}{V_i})$ $W = 2R(300)\ln(\frac{2V_i}{V_i})$ $W = 3458J$	$\Delta U = 0$ $>> \text{Why}? <<$
T->2T Adiabatic (Q=0)	Q=0	$W = -\Delta U$ $W = -7483J$	$\Delta U = nc_v(\Delta T)$ $\Delta U = 3R(600-300)$ $\Delta U = 7483J$

(3) Suppose that 2 moles of an IDG go through the following process:
(1) T_i=300K, T_f=600K adiabatic process.
(2) V_i=0.05m^3, V_f=0.10m^3 isothermal process
(3) T_i=600K, T_f=300K, adiabatic process
(4) V_i=0.10m^3, V_f=0.05m^3 isothermal process

Calculate the following quantities:
(a) The total heat added to the process
(b) The total heat evolved by the process
(c) The net heat added to (or evolved from) the system.
(d) The work done by the system along the entire process.
(e) ΔU for this entire closed process.
(f) Define the engine's efficiency by:

$$\varepsilon \equiv \frac{W}{Q_{input}}$$

Find out how efficient this engine is.
(a) Heat is added to the system or removed from the system only along the isothermal portions of the process ***because along the adiabatic portions, no heat enters or leaves the system***. Thus, for process (2 and 4) we have (as in the second problem):

$$W_{(2)} = nRT\ln\left(\frac{V_f}{V_i}\right) = 2(8.314)(600)\ln\left(\frac{0.1}{0.05}\right) = 6915J$$

$$W_{(4)} = nRT\ln\left(\frac{V_f}{V_i}\right) = 2(8.314)(300)\ln\left(\frac{0.05}{0.1}\right) = -3457.7J$$

Thus
Q(1)=0
Q(2)=6915.4J (answer to (a))
Q(3)=0
Q(4)=-3457.7J (answer to (b))
The total heat added to the system is:

$$Q=Q(2)+Q(3)=6915.4J-3457.7J=3457.7J \text{ (answer to (c))}$$

We can now calculate the total work done:

$$W=6915.4J-3457.7J=+3457.7J$$

(this is the answer to (d))
We can now calculate ΔU for this process:

$$\Delta U = Q - W = 3457.7 - 3457.7 = 0J$$

Whew! this had to be the case (why?)
The efficiency is now easily given by:

$$\varepsilon = \frac{3457.2}{6915} = 0.50 \Rightarrow 50\% \text{ efficiency}$$

(4) Show for the closed process involving 2 isotherms and 2 adiabatic processes as in problem (3) that the efficiency is given by:

$$\varepsilon_c = 1 - \frac{T_{cold}}{T_{hot}}$$

Since for any closed process involving the IDG, ΔU must equal 0 (why?)
We have from the 1st law:

$$\Delta U = Q - W \Rightarrow Q_{input} - Q_{rejected} = W$$

This is usually written in terms of Q_H and Q_c (h-hot, c-cold) because it is observed that heat flows from hotter regions towards colder regions. Thus, the heat input to the system must have come from a hot reservoir and was rejected to a colder reservoir.
If the first law is valid, then we can write the efficiency defined in problem 3 as:

$$\varepsilon = \frac{W}{Q_{input}} = \frac{Q_H - Q_c}{Q_H} = 1 - \frac{Q_c}{Q_H}$$

We can check real easily that this is true for problem (3):

$$\varepsilon_{problem2} = 1 - \frac{3457.7}{6915.4} = 1 - 0.5 = 0.5 \Rightarrow 50\%$$

(not all efficiencies come out to be 50% so don't fall into that trap).
In more general terms, for a cycle such as this, we have:

$$Q_H = nRT_H \ln\left(\frac{V_{1,f}}{V_{1,i}}\right)$$

and

$$Q_c = nRT_c \ln\left(\frac{V_{2,f}}{V_{2,i}}\right)$$

We can easily find the ratio of these:

$$\frac{Q_c}{Q_H} = \frac{nRT_c \ln\left(\frac{V_{2,f}}{V_{2,i}}\right)}{nRT_H \ln\left(\frac{V_{1,f}}{V_{1,i}}\right)}$$

From the adiabatic process, we have:

$$T_H V_{1,f}^{\gamma-1} = T_c V_{2,i}^{\gamma-1} \Rightarrow V_{2,f} = \left(\frac{T_H}{T_C}\right)^{\frac{1}{\gamma-1}} V_{1,i} \quad \text{and} \quad T_C V_{2,f}^{\gamma-1} = T_H V_{1,i}^{\gamma-1} \Rightarrow V_{2,i} = \left(\frac{T_H}{T_C}\right)^{\frac{1}{\gamma-1}} V_{1,f}$$

So:

$$\ln\left(\frac{V_{2,f}}{V_{2,i}}\right) = \ln\left(\frac{\left(\frac{T_H}{T_C}\right)^{\frac{1}{\gamma-1}} V_{1,i}}{\left(\frac{T_H}{T_C}\right)^{\frac{1}{\gamma-1}} V_{1,f}}\right) = \ln\left(\frac{V_{1,i}}{V_{1,f}}\right)$$

$$\Rightarrow \frac{Q_c}{Q_H} = \frac{T_c}{T_H} \frac{\ln\left(\frac{V_{1,i}}{V_{1,f}}\right)}{\ln\left(\frac{V_{1,i}}{V_{1,f}}\right)} = \frac{T_c}{T_H}$$

Thus, the efficiency is given by:

$$\varepsilon_c = 1 - \frac{T_c}{T_H}$$

Again, you can check this for problem (2) to easily get 50% efficiency. This cycle which I have described is the Carnot cycle and it is the most efficient cycle which can be obtained. This will be shown later. Important to note, however, is that this last form of the efficiency ***ONLY is valid for a Carnot cycle***! That's why that little "c" appears beside the ε symbol.

(5) What is the efficiency of a Carnot cycle which is operating between the temperature extremes of 0^0C and -200 ^{0}C.

Solution:

We showed in problem (4) that for a Carnot cycle,

$$\varepsilon_c = 1 - \frac{T_c}{T_H}$$

It is possible to show by considering to exhaustion all possible combinations of isotherms-adiabats that all Carnot cycles have this same efficiency. This, in fact, is the second law of thermodynamics.

What we have not yet shown is that this is the most efficient cycle possible. That needs to wait till we talk about Entropy. I prefer to show this using entropy considerations.

The solution to our present problem:

$$\varepsilon_c = 1 - \frac{T_c}{T_H} = 1 - \frac{-200}{0} = \infty$$

Oops! if you answer this type of question like this, then you've fallen into my trap and missed the problem. I usually don't even think about giving partial credit for this type of error … that's why you want to make sure you don't make this mistake.

The problem is this: you must work with your temperatures in K only!

The real solution:

$$\varepsilon_c = 1 - \frac{T_c}{T_H} = 1 - \frac{73.15}{273.15} = 0.732 \Rightarrow 73.2\% \text{ efficient}$$