| | Physics 240: Worksheet 22 | Name: ________________ |
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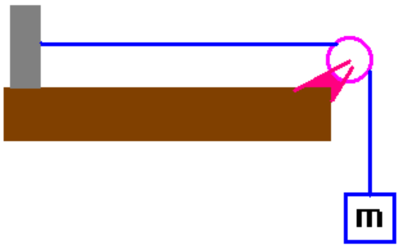
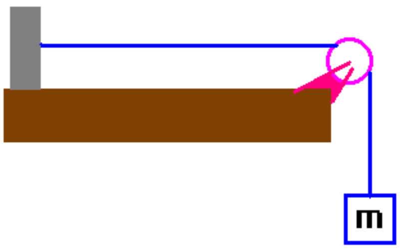
(1) A string has a total length of 5 m and a total mass of 0.01 kg. If the string has a tension of 10N applied to it, what is the speed of a transverse wave on this string?




(2) A string of length 3 m and total mass 0.001 kg is connected to a mass of 15 kg as shown. Calculate the speed of a transverse wave on this string.

(3) A wire is 5000 m long and has a total mass of 1 kg. The wire is under a tension of 30N. The wire is attached between two large trees. How long will it take for pulse which is an "upward" pluck to travel there and back and what will the returning pulse look like?




(4) A string of length 10 m and total mass 0.001 kg is connected to a mass m as shown. Suppose that this string has a very high elastic limit (meaning that it takes lots of pressure on the string till it will stretch very much). How must mass must you place on the string in order to produce a wave speed of 2000 m/s.

(5) Suppose a wire has a mass per unit length of 1kg/m. What is the speed of propagation that would exist if this wire were hung between two poles and was under a tension of 100 N? How long would it take for a pulse to travel between two telephone poles (holding this wire) which were 40 m apart?

## Types of waves

There are two types of wave that we will study: transverse and longitudinal waves.
Not all waves fit into these two categories (for example, water waves are both types).
Soon, we'll also study harmonic traveling waves. These will be waves that occur due to repetitive motion, not just an isolated impulse on a medium.

The distinguishing characteristic here is this:
For transverse waves, the ***disturbance is perpendicular*** to the direction of wave propagation. These types of waves include pulses on a string, and electromagnetic waves.

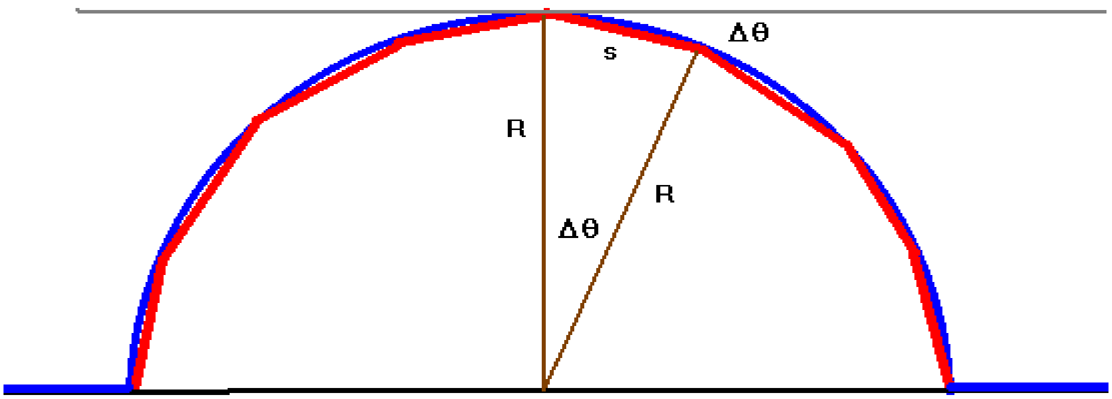
For longitudinal waves, the ***disturbance amplitude is parallel or antiparallel*** to the direction of wave propagation. These types of waves include sound waves, waves on a slinky and jerks along the length of a string.

We're going to study as our model transverse waves first. Probably the easiest way to study longitudinal waves is by imagining that they are transverse waves, which we will do when we discuss sound.

## Characteristics of transverse waves:

These types of waves are characterized by two types of velocities:
(1) the wave speed (v) which answers the question how fast energy is being transported
(2) the transverse velocity which tells you how fast an element of the wave is approaching or departing from the equilibrium position (it is a velocity which is transverse to the wave speed).

**Wave speed of a transverse pulse on a string**

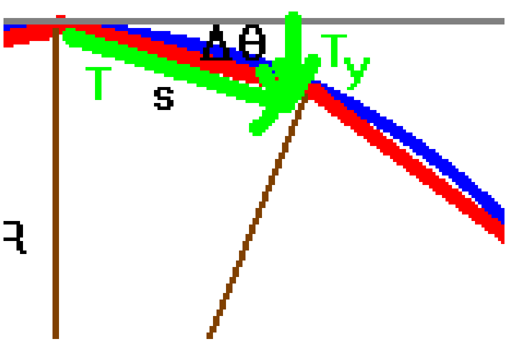



This picture shows an approximation of a pulse on a string as a series of straight line segments. I've purposely chosen the length of the segments so that the two angles indicated are indeed here the same. The length of the segment shown is given by:

$$s \approx R(\Delta\theta)$$

And this segment has a mass given by

$$m = \mu(s) = \mu R(\Delta\theta)$$




This picture show an enlargement of the apex of the pulse. The restoring force is the y component of the tension which is given by:

$$T_y = T\sin(\theta) \approx T(\Delta\theta)$$

According to Newton's law, then

$$F = ma \Rightarrow T(\Delta\theta) = \mu R(\Delta\theta)\left(\tfrac{v^2}{R}\right) \Rightarrow T = \mu v^2$$

We can solve this for the velocity as:

$$v = \sqrt{\frac{T}{\mu}}$$

***Notice that the wave speed depends only on T and $\mu$ for linear transverse oscillations!***

**Description of the pulse**

The propagation of a pulse on an infinite string is easily described. The function is:

$$y(x,t) = f(x \pm vt)$$

Now, you need to make a decision as to which of these describes the wave under study.

We do this by using the "method of constant phase."

We imagine we're walking along side a pulse on a string. Then, as far as the co-moving observer is concerned, the disturbance looks like:

$$y(x,t) = f(\varphi)$$

where $\varphi$ is a constant. Since this phase does not change, we thus have:

$$\varphi = x \pm vt \Rightarrow \frac{d\varphi}{dt} = \frac{dx}{dt} \pm v\frac{dt}{dt} \Rightarrow 0 = \frac{dx}{dt} \pm v$$

This is easily solved to give:

$$\frac{dx}{dt} = \mp v$$

This means that if the wave is traveling in the +x direction, it is described by

$$y(x,t) = f(x - vt)$$

and if the wave is traveling in the –x direction, it is described by

$$y(x,t) = f(x + vt).$$

I've made an animated gif to show this and also I have made an excel spreadsheet that you can play with to watch the pulses change position (you change the time t on the spreadsheet).

## Calculation of the transverse velocity

The transverse velocity is given by

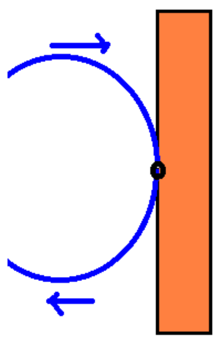
$$v_t = \frac{dy}{dt}\Big|_{x \text{ constant}}$$

Normally, this is quite difficult for non-calculus students (=impossible). Calculus students will have an easier time of this, but there are still some complications which come up (you need to take derivative at a fixed x). You might imagine that the type of derivative I'm using here is not actually telling the whole story. Basically, you need to treat x as if it were a constant when you take that derivative. The type of derivative that I'm beating around the bush on here is actually a partial derivative, but you only run into those in Calculus 3 so I'm skirting the issue here a bit. Don't let that stop you.
However, for those of you that do understand partial derivatives, the transverse velocity is given by:

$$v_t = \frac{\partial y(x,t)}{\partial t}$$

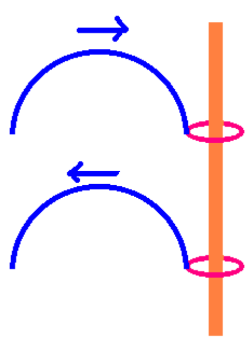
## Boundary conditions on waves

We'll consider only two types of boundary conditions here, namely fixed and free boundaries. Other types of boundaries are possible but we're going to stick with these two limiting cases here.



### (1) Fixed boundary conditions

Imagine that a string has one end tied to a wall. Then an impulse on the sting will pull up on the wall. The wall responds by pulling down on the string and this "down" pull is then transmitted back along the string. What was an "up" pulse will be reflected as a "down" pulse. You'd say that there was a $180^0$ phase shift in the reflected wave here.



### (2) Free boundary conditions

Imagine that a string is connected to a massless ring on a rod. Then an impulse on the string will pull up on the ring, and the ring will respond by moving up. As the pulse passes through the ring, the ring will be pulled back down. The pulse will be reflected so that the outgoing phase is the same … what was an "up" pulse will be reflected back as an "up" pulse. You'd say that there was a $0^0$ phase shift in the reflected wave here.

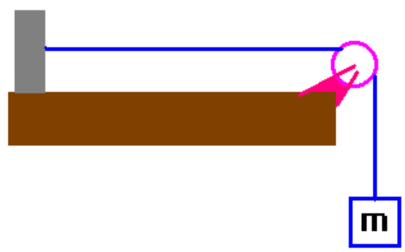
There are some additional details for these boundary conditions regarding the area of the pulses that we won't go into at the present time.

(1) A string has a total length of 5 m and a total mass of 0.01 kg. If the string has a tension of 10N applied to it, what is the speed of a transverse wave on this string?

Solution:

$v = \sqrt{\frac{T}{\mu}}$. Here, we will need to calculate $\mu$. This is given by: $\mu = \frac{m}{L} = \frac{0.01\text{kg}}{5\text{m}} = 2\text{x}10^{-3}\text{kg/m}$. Thus, we can now easily find the wave speed:

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{10\text{N}}{2\text{x}10^{-3}\text{kg/m}}} = \sqrt{5000} = 70.7\,\frac{\text{m}}{\text{s}}$$



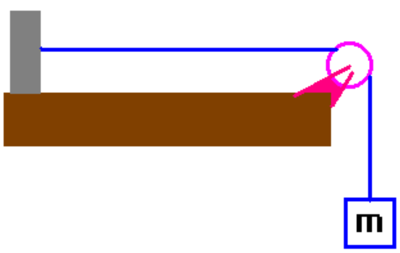

(2) A string of length 3 m and total mass 0.001 kg is connected to a mass of 15 kg as shown. Calculate the speed of a transverse wave on this string.

Solution:

The linear mass density of the string is given by $\mu = \frac{m}{L} = \frac{0.001\text{kg}}{3\text{m}} = 3.33\text{x}10^{-4}\text{kg/m}$. The speed of a transverse pulse on this string is then easily obtained once the tension is known. The tension is given by $T = mg = 15(9.8) = 147\text{N}$. The wave speed is then given by: $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{147\text{N}}{3.33\text{x}10^{-4}\text{kg/m}}} = 664\,\frac{\text{m}}{\text{s}}$.

(3) A wire is 5000 m long and has a total mass of 1 kg. The wire is under a tension of 30N. The wire is attached between two large trees. How long will it take for pulse which is an "upward" pluck to travel there and back and what will the returning pulse look like?

Solution:
As usual, the wave speed is given by $v = \sqrt{\frac{T}{\mu}}$ where $\mu = \frac{m}{L} = \frac{1kg}{5000m} = 2x10^{-4}\,\frac{kg}{m}$. The wave speed is then $v = \sqrt{\frac{30N}{2x10^{-4}N/m}} = 387\frac{m}{s}$. The pulse travels through a total distance of 10000m so we can find the time that it takes for the pulse to come back: $x = vt \Rightarrow t = \frac{x}{v} = \frac{10000}{387} = 25.8s$
Notice that I have purposely used T and t in this problem. You must understand the meaning of the symbol in the context in which it is used!
The boundary condition is fixed. If the outgoing pulse was an "up" pulse, the reflected pulse will be a "down" pulse when it comes back.




(4) A string of length 10 m and total mass 0.001 kg is connected to a mass m as shown. Suppose that this string has a very high elastic limit (meaning that it takes lots of pressure on the string till it will stretch). How must mass must you place on the string in order to produce a wave speed of 2000 m/s.

Solution: the linear mass density here is $\mu = \frac{M}{L} = \frac{0.001kg}{10m} = 1x10^{-4}\,\frac{kg}{m}$. The wave speed is given by $v = \sqrt{\frac{T}{\mu}} \Rightarrow T = v^2\mu = mg \Rightarrow m = \frac{v^2\mu}{g}$. Thus, the required mass to produce such a large wave speed is $m = \frac{\left(2000\frac{m}{s}\right)^2\left(1x10^{-4}\frac{kg}{m}\right)}{9.8\frac{m}{s^2}} = 40.8kg$.

(5) Suppose a wire has a mass per unit length of 1kg/m. What is the speed of propagation that would exist if this wire were hung between two poles and was under a tension of 100 N? How long would it take for a pulse to travel between two telephone poles (holding this wire) which were 40 m apart?
Solution: The wire would not actually hang particularly straight between the poles: in fact, it would form a catenary (look at the unrivaled arch in Saint Louis while standing on your hands sometime … it's an inverted catenary). I am ignoring this here. The wave speed is $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{100N}{1kg/m}} = 10\frac{m}{s}$. It will take a time $t = \frac{d}{v} = \frac{40m}{10m/s} = 4s$ to travel this distance. Of course, if you look at the actual shape of the curve, it is going to be a bit different … this is the least amount of time for this situation.