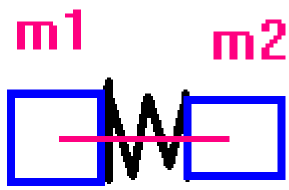
	Physics 240: Worksheet 13	Name: ________________

(1) It's a rainy day at the parking lot and two cars (m_1=$2m_2$) have a collision. Car 2 was not moving and had its breaks on and was knocked for 10 m at an angle (negative) of 35^0 from an arbitrarily placed bisector. The driver of the first car immediately applied the brakes and was diverted for 20m before stopping at an angle (positive) of 35^0 from the bisector. If the coefficient of friction between the wet road and the cars is μ=0.5, how fast was the first car moving before the collision?

(2) Two masses, m_1=1.0 kg and m_2=2.0 kg are connected by a massless spring. A string is also holding the two masses. The string is cut. It is observed that mass m2 moves off with a speed of 2.3 m/s to the right. What is the velocity of mass m1?



(3) A special operations soldier (m=150 kg) covertly jumps from a bridge (which is 2 m above a river) onto a speed boat (m=400 kg) which is moving up river at a speed of 5 m/s. How fast is the boat moving afterwards?

(4) An astronaut (m=175 kg) working outside the ISS becomes disteathered from the station and has only a hammer (m=25 kg) to work with. Suppose the astronaut is initially at 10 m from the ISS and is stationary with respect to the ISS. How fast must the astronaut throw the hammer in order to reach the ISS in 10 s?

(5) A soccer player can apply a force of 30 N to a ball of mass m=1.5 kg. If the player remains in contact with the ball for 0.7 s, how fast will the ball be moving when it leaves the player's foot?

This problem is very similar to the last problem on worksheet 12, only it is different.

(1) It's a rainy day at the parking lot and two cars (m_1=2m_2) have a collision. Car 2 was not moving and had its breaks on and was knocked for 10 m at an angle (negative) of 35^0 from an arbitrarily placed bisector. The driver of the first car immediately applied the brakes and was diverted for 20m before stopping at an angle (positive) of 35^0 from the bisector. If the coefficient of friction between the wet road and the cars is μ=0.5, how fast was the first car moving before the collision?

We can find out how fast the second car was traveling immediately after the collision: $\Delta K_{NC} = \Delta K_c \Rightarrow -\mu mgx = -\frac{1}{2}mv^2 \Rightarrow v = \sqrt{2\mu gx}$ where I've assume a positive velocity. We can now answer questions about the initial velocity of the first car by finding the components of the final velocities. **We only really know for sure, in this collision, that the momentum was conserved at the moment of impact.**

For this situation, the velocity of the second car right after the collision was 9.899 m/s.

How? $v = \sqrt{2\mu gx} = \sqrt{2(.5)(9.8)(10)} = 9.899\,\frac{m}{s}$

The momentum components are: $\begin{matrix} P_{x,2} = m_2 v_{x,2} = m_2(9.899)\cos(-35) = 8.109m_2 \\ P_{y,2} = m_2 v_{y,2} = m_2(9.899)\sin(-35) = -5.678m_2 \end{matrix}$.

We can, in the same way, find out how fast the first car was moving after the collision, namely at 8.85 m/s.

How? $v = \sqrt{2\mu gx} = \sqrt{2(.5)(9.8)(20)} = 14.000\,\frac{m}{s}$

The components of momentum for the first car are:

$$P_{x,1} = m_1 v_{x,1} = m_1(14)\cos(35) = 11.47m_1$$
$$P_{y,1} = m_1 v_{y,1} = m_1(14)\sin(35) = 8.030m_1$$

Since momentum is conserved, we then have:

$$P_{x,1,before} = P_{x,1,after} + P_{x,2,after}$$
$$P_{y,1,before} = P_{y,1,after} + P_{y,2,after}$$

This is where the problem now is different from the previous problem:

$$P_{x_{before}} = P_{x_{after}} \Rightarrow m_1 v_{x,1_{before}} = m_1 v_{x,1_{after}} + m_2 v_{x,2_{after}}$$
$$\Rightarrow v_{x,1_{before}} = v_{x,1_{after}} + h v_{x,2_{after}} ;\text{using } m_2 = hm_1$$
$$P_{y_{before}} = P_{y_{after}} \Rightarrow m_1 v_{y,1_{before}} = m_1 v_{y,1_{after}} + m_2 v_{y,2_{after}}$$
$$\Rightarrow v_{y,1_{before}} = v_{y,1_{after}} + h v_{y,2_{after}} ;\text{using } m_2 = hm_1$$

We thus have:

$$v_{x,1_{before}} = v_{x,1_{after}} + h v_{x,2_{after}} \Rightarrow v_{x,1_{before}} = 11.47 + h(8.109)$$
$$v_{y,1_{before}} = v_{y,1_{after}} + h v_{y,2_{after}} \Rightarrow v_{y,1_{before}} = 8.030 + h(-5.678)$$

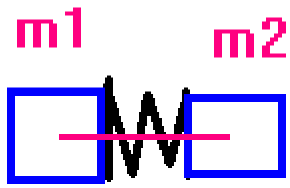
Thus we can find the velocity before the collision:

$$v_{1,before} = \sqrt{[11.47 + h(8.109)]^2 + [8.030 + h(-5.678)]^2}$$

In the present case, h=0.5 so we have the result:

$$v_{1,before} = \sqrt{[15.5245]^2 + [5.191]^2} = 16.4\,\frac{m}{s}$$

(2) Two masses, m_1=1.0 kg and m_2=2.0 kg are connected by a massless spring. A string is also holding the two masses. The string is cut. It is observed that mass m_2 moves off with a speed of 2.3 m/s to the right. What is the velocity of mass m_1?



Solution: The spring is internal to this system. It's going to supply the needed kinetic energy to speed the individual masses up after everything's over. The conservation of momentum thus applies to this system. $(m_1 + m_2)v_{before} = m_1 v_{1,after} + m_2 v_{2,after}$. Since the system is initially not moving, the left side of this vanishes. Thus,

$$0 = m_1 v_1 + m_2 v_2 \Rightarrow v_1 = -\frac{m_2}{m_1} v_2 = -\frac{2}{1} 2.3 = -4.6 m/s.$$

The conservation of kinetic energy does not apply here because this is basically an inelastic collision in reverse.

(3) A special operations soldier (m=150 kg) covertly jumps from a bridge (which is 2 m above a river) onto a speed boat (m=400 kg) which is moving up river at a speed of 5 m/s. How fast is the boat moving afterwards?

Solution: The special ops soldier has a momentum in the -y direction upon striking the boat. The velocity is given by energy considerations:

$$\Delta K_{nc} = \Delta K + \Delta U = 0 \Rightarrow \tfrac{1}{2} m_s v_s^2 - m_s g h = 0 \Rightarrow v_s = \sqrt{2gh} = \sqrt{2(9.8)2} = 6.26 \tfrac{m}{s}$$

Momentum is conserved in the collision but not kinetic energy since it is completely inelastic. Thus:

$$\vec{P}_{before} = \vec{P}_{after} \Rightarrow m_s \vec{v}_s + m_b \vec{v}_{b,before} = (m_s + m_b)\vec{v}_{after}$$

It's going to be easier to separate this into components.

$$\vec{v}_s = 0\hat{i} - 6.26\hat{j}; \vec{v}_{b,before} = 5\hat{i} + 0\hat{j}; \vec{v}_{after} = v_{x,after}\hat{i} + v_{y,after}\hat{j}$$

We thus have:

$$150(-6.26) + 400(0) = (550)(v_{y,after}) \Rightarrow v_{y,after} = -1.71 \tfrac{m}{s}$$

$$150(0) + 400(5) = 550(v_{x,after}) \Rightarrow v_{x,after} = 3.64 \tfrac{m}{s}$$

The velocity vector of the combination after the collision is thus:

$$\vec{v}_{after} = \left[3.64\hat{i} - 1.71\hat{j}\right]\tfrac{m}{s} \Rightarrow |\vec{v}| = \sqrt{(3.64)^2 + (1.71)^2} = 4.02 \tfrac{m}{s}$$

Notice that if you ignored the y-velocity component, the boat would move at 3.64 m/s afterwards.

(4) An astronaut (m=175 kg) working outside the ISS becomes disteathered from the station and has only a hammer (m=25 kg) to work with. Suppose the astronaut is initially at 10 m from the ISS and is stationary with respect to the ISS. How fast must the astronaut throw the hammer in order to reach the ISS in 10 s?
Solution: We need to find the required velocity which is 10m/10s=1 m/s for the astronaut. We'll apply conservation of momentum here to find the speed of the hammer:

$$m_h v_h + m_a v_a = 0 \Rightarrow v_h = -\tfrac{m_a}{m_h} v_a = -\tfrac{175}{25} 1 = -7 m/s.$$

Calculus version: (5) A soccer player can apply a force of 30 N to a ball of mass m=1.5 kg. If the player remains in contact with the ball for 0.7 s, how fast will the ball be moving which it leaves the player's foot?
Solution: This problem introduces a new quantity, impulse. The definition for impulse is given by $\vec{J} \equiv \vec{F}(\Delta t)$. Let's see what this results in. From Newton's law,

$$\text{Non-calculus version: } \vec{F} = m\tfrac{\Delta\vec{v}}{\Delta t} \Rightarrow \vec{F}\Delta t = m(\Delta\vec{v}) \Rightarrow \vec{J} = \Delta\vec{P}$$

$$\text{Calculus version:} \vec{F} = m\tfrac{d\vec{V}}{dt} \Rightarrow \int \vec{F}dt = \int d\vec{P} = \Delta\vec{P}.$$

The point is that an impulse is the thing that produces a change in momentum. For this problem then, $J = 30(0.7) = 21Ns \Rightarrow \Delta v = \tfrac{J}{m} = \tfrac{21}{1.5} = 14 m/s$
Now, if the force varies over some interval, you'll need to add up the individual impulses to get the total change in momentum. This looks like:

$$\text{Non-calculus version: } \Delta\vec{P} = \sum \vec{J}$$

$$\text{Calculus version:} \Delta\vec{P} = \int_0^t \vec{F}(t)dt$$

For example, suppose

$$\vec{F}(t) = bt\hat{i}$$

where b has units of N/s and the expression represents a linear increase in force with time. The total impulse delivered is then

$$\vec{J} = \int_0^t bt dt \hat{i} = \tfrac{bt^2}{2}\Big|_0^t \hat{i} = \tfrac{bt^2}{2}\hat{i} = \Delta\vec{P}$$

Non-calculus version: (5) A soccer player can apply a force of 30 N to a ball of mass m=1.5 kg. If the player remains in contact with the ball for 0.7 s, how fast will the ball be moving which it leaves the player's foot?

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