	Physics 240:Worksheet 04	Name ________________

(1) A ball is thrown from the top of a building (50 m high) with an initial velocity vector given by $\vec{V}_i = 5\hat{i} + 0\hat{j}$. When the ball strikes the ground, how far in the x-direction did it move?

(2) Suppose one of my faster cats is chasing a mouse (which is even faster). My cat leaps into the air at a 33^0 angle. If my cat was initially moving at 10 m/s, (a) what was my cat's range and (b) how long was my cat in the air?

(3) If a ball is thrown up at some angle, what must that angle be so that the maximum range results when the ball returns to the initial level?

(4) A vendor on a train moving in the forward direction at 2.00 m/s pushes her cart toward the rear of a train at 0.47 m/s while an ant on a sandwich crawls toward the front of the train at 0.01 m/s. What is the velocity of the train station with respect to the ant? Call the direction in which the train is moving relative to the train station the +x direction here.

(5) A person looking out the window of a stationary train notices that raindrops are falling vertically down at a speed of 3.0 m/s relative to the ground. When the train moves at a constant velocity, the raindrops make an angle of 30^0 when they move past the window. How fast is the train moving?

(1) A ball is thrown from the top of a building (50 m high) with an initial velocity vector given by $\vec{V}_i = 5\hat{i} + 0\hat{j}$. When the ball strikes the ground, how far in the x-direction did it move?
Solution:
You need to find the time of fall. This can most directly be found from: $y = y_i + v_{y,i}t + \frac{1}{2}at^2$. This time is then used to find the x-displacement. So, $0 = 50 - 4.9t^2 \Rightarrow t = \pm\sqrt{\frac{50}{4.9}} = \pm 3.19s$. The physical solution here is t=+3.19s. The x-displacement is then given by $x = v_{x,i}t = 5(3.19) = 16m$.

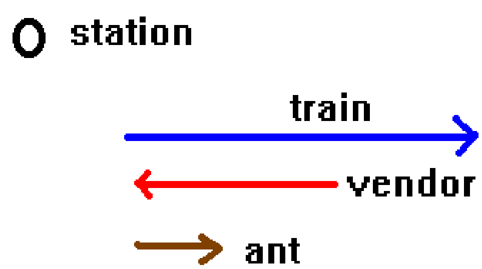
(2) Suppose one of my faster cats is chasing a mouse (which is even faster). My cat leaps into the air at a 33^0 angle. If my cat was initially moving at 10 m/s, (a) what was my cat's range and (b) how long was my cat in the air?
Solution:
Let's obtain a general expression for the range: for a projectile with an initial velocity vector given by $\vec{V}_i = V_{x,i}\hat{i} + V_{y,i}\hat{j}$, we can find the time by applying the second equation of motion: $v_y = v_{y,i} - gt$. So long as we are talking about level ground, the cat will return to the same level at the end as at the beginning. Thus, we are solving for the time when the y velocity is $-V_{y,i}$ or: $-v_{y,i} = v_{y,i} - gt \Rightarrow 2v_{y,i} = gt \Rightarrow t = \frac{2v_{y,i}}{g}$. Now use this in the x-equation to find the range: $x = x_i + v_{x,i}t \Rightarrow \Delta x = v_{x,i}\frac{2v_{y,i}}{g} = \frac{2v_{x,i}v_{y,i}}{g}$. Now, we can apply trigonometry: $v_{x,i} = |\vec{V}_i|\cos(\theta)$ and $v_{y,i} = |\vec{V}_i|\sin(\theta)$. Thus, (we'll let R=$\Delta$x): $R = \frac{2v_i^2\cos(\theta)\sin(\theta)}{g} = \frac{v_i^2\sin(2\theta)}{g}$.We'll use this in the next problem.
Now, $t = \frac{2v_{y,i}}{g} = \frac{2v\sin(\theta)}{g} = \frac{2(10)\sin(33)}{9.8} = 1.11s$ and $R = \frac{v_i^2\sin(2\theta)}{g} = \frac{10^2\sin(66)}{9.8} = 9.32m$

(3) If a ball is thrown up at some angle, what must that angle be so that the maximum range results when the ball returns to the initial level? What is this maximum range?
Solution:
In the previous problem, we obtained the result for the range at any angle: $R = \frac{v_i^2\sin(2\theta)}{g}$. We want to maximize this result. Since the maximum will be when $\sin(2\theta) = 1$, we then find that the angle for maximum range is θ=45^0. The maximum range is then given by $R = \frac{v^2}{g}$.

(4) A vendor on a train moving in the forward direction at 2.00 m/s pushes her cart toward the rear of a train at 0.47 m/s (relative to the train) while an ant on a sandwich crawls toward the front of the train at 0.01 m/s (relative to the vendor). What is the velocity of the train station with respect to the ant? Call the direction in which the train is moving relative to the train station the +x direction here.
Solution:
Classically (but not at speeds close to the speed of light), velocities add according to $\vec{V}_{fixed} = \sum \vec{V}_{moving}$. It is probably important to place another restriction on this problem here: even though each velocity might be low, if enough of the systems are added in this way, relativistic considerations may need to be considered also. Here, the station corresponds to the easiest fixed frame to talk about, although the ant's frame is really where we should be looking. What does the ant see?
In any event, according to an observer in the station, the observer would see

$$v_{ant} = -2 + 0.47 - .01 = -1.54 \tfrac{m}{s}$$

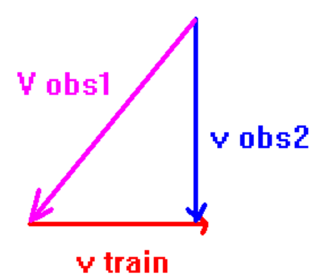
The ant sees the station moving in the +x direction with v=+1.54 m/s.



Velocity of (element labeled "A") as seen by (element labeled "B"): note symmetry here.

	A: Station	A: Train	A: Vendor	A: Ant
B: Station	0	=-2	=-2+0.47=-1.53	=-2+0.47-0.01=-1.54
B: Train	=+2	0	=+0.47	=+0.47-0.01=+0.46
B: Vendor	=+2-0.47=+1.53	=-0.47	0	=-0.01
B: Ant	=+2-.47+.01=+1.54	=-0.47+0.01=-0.46	=+0.01	0

(5) A person (obs2) looking out the window of a stationary train notices that raindrops are falling vertically down at a speed of 3.0 m/s relative to the ground. When the train moves at a constant velocity (v_{train}), the raindrops make an angle of 30^0 when they move past the window (obs1). How fast is the train moving?



Solution: Consider the vector sketch shown. The observer in the train sees the "purple" line marked vobs1. The fixed observer sees the "blue" line marked vobs2. The train is moving forward with v_{train}. The angle between vobs2 and vobs1 is given by $\tan(\theta)=\frac{v_{train}}{v_{obs2}} \Rightarrow v_{train}=V_{obs2}\times\tan(\theta)=3\times\tan(30)=1.7\frac{m}{s}$. If you wanted to do this with vectors, here is the way: the observer on the train sees obs2 moving in the –x direction with v_{train}. Thus, according to the classical velocity addition: $\vec{v}_{obs1}=\vec{v}_{train}+\vec{v}_{obs2}=-|\vec{v}_{train}|\hat{i}-3\hat{j}.$. We can then find the train velocity from: $\tan(\theta)=\frac{v_{train}}{v_{obs2}}$.

Let me show you a couple more examples of how this might work. Suppose a river is flowing with a velocity vector given by: $\vec{v}_{stream}=+5\hat{i}\frac{m}{s}$. A boat in still water can move at only a speed of 10 m/s (except, of course, at which time momentarily any speed is possible). If the captain of the boat wants to cross the river and land at a point on the opposite shore directly across from the starting point, at which angle must the captain direct the boat relative to the y- axis?

By the statement of the problem, I am assigning the positive x direction to be in the direction that water is flowing, parallel to the shore.

In this case, you need to require that the total velocity of the boat on the river is only in the y-direction. (hmm… it looks a lot harder till you see that point). Then let's find the various velocities:

In still water, the boat has a velocity vector given by:

$$\vec{v}_{boat}=v_{x,boat}\hat{i}+v_{y,boat}\hat{j}=10\cos(\theta_x)\hat{i}+10\sin(\theta_x)\hat{j}$$

Where θ_x is the angle of the boat's velocity vector (in still water) relative to the positive x direction.

The river has a velocity vector given by:

$$\vec{v}_{river}=5\hat{i}+0\hat{j}$$

The total boat velocity vector (relative to an observer on the shore) given by:

$$\vec{v}_{boat,obs}=\vec{v}_{boat}+\vec{v}_{river}=\left[5+10\cos(\theta_x)\right]\hat{i}+10\sin(\theta_x)\hat{j}$$

Now to go directly across the river, we require no net x velocity. Thus:

$$5+10\cos(\theta_x)=0\Rightarrow\cos(\theta_x)=-\frac{5}{10}=-0.5\Rightarrow\theta_x=120^0$$

Relative to the positive y axis, the angle would be then:

$$\theta_x-90=120-90=30^0$$

These types of problems can be more difficult until you ask the right question. Let me show you what I mean. Suppose the river is 100 m wide. At what angle must the ship be directed in order to dock at a port which has coordinates $(x,y)=(1000,100)$?

In this case, you need to include an additional equation involving the time required to cross the river, and so we have for the velocity vector again:

$$\vec{v}_{boat,obs} = \vec{v}_{boat} + \vec{v}_{river} = \left[5 + 10\cos(\theta_x)\right]\hat{i} + 10\sin(\theta_x)\,\hat{j}$$

But now, you require the following additional constraint:

$$\vec{R} = x\hat{i} + y\hat{j} = \left[5 + 10\cos(\theta_x)\right]t\,\hat{i} + \left[10\sin(\theta_x)\right]t\,\hat{j} = 1000\hat{i} + 100\hat{j}$$

This is actually two simultaneous equations to solve. The requirement is that at time t, the boat is located at the coordinates (1000,100). This then gives the two equations:

$$t = \frac{100}{10\sin(\theta_x)} = \frac{10}{\sin(\theta_x)} \text{ and } t = \frac{1000}{\left[5+10\cos(\theta_x)\right]}$$

We eliminate time to give:

$$\frac{1000}{\left[5+10\cos(\theta_x)\right]} = \frac{10}{\sin(\theta_x)} \Rightarrow 1000\sin(\theta_x) = 50 + 100\cos(\theta_x)$$

$$\Rightarrow 100\sin(\theta_x) = 5 + 10\cos(\theta_x) \Rightarrow \sin(\theta_x) = 0.05 + 0.1\cos(\theta_x)$$

Use this now:

$$x \equiv \cos(\theta) \Rightarrow \sin(\theta) = \pm\sqrt{1 - x^2}$$

And the equation becomes:

$$1 - x^2 = \left[0.05 + 0.1x\right]^2 \Rightarrow 1 - x^2 = 0.0025 + 0.01x + 0.01x^2$$

$$\Rightarrow x^2(1.01) + x(0.01) - 0.9975 = 0$$

$$x = \frac{-.01 \pm \sqrt{(.01)^2 + 4(1.01)(0.9975)}}{2(1.01)} = \frac{-.01 \pm \sqrt{.0001 + 4.0299}}{2.02} = \frac{-.01 \pm \sqrt{4.03}}{2.02} = \begin{matrix} 0.9889 \\ -0.9988 \end{matrix}$$

Now solve this for the angle:

$$\theta = \begin{matrix} +9.49^0 \\ +197^0 \end{matrix}$$

The valid solution is the first solution. You can check this quickly by approximately solving the problem as:

$$x^2(1.01) = .9975 \Rightarrow x = \pm 0.9938 \Rightarrow \theta = 7.1^0$$

Which is close to the solution above. This can also be solved with a spreadsheet to give the same answer.