

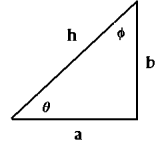
(1) Which of the following equations are dimensionally correct? The following quantities have the following dimensions: {L=length, M=mass, t=time}

$$F: \left[\frac{ML}{t^2} \right]; m: [M]; a: \left[\frac{L}{t^2} \right]; t: [t]; x: [L]; E: \left[\frac{ML^2}{t^2} \right]; v: \left[\frac{L}{t} \right]; s: [L]$$

- (a) $F=ma$
- (b) $x = \frac{1}{2}at^3$
- (c) $E = \frac{1}{2}mv$
- (d) $E=max$
- (e) $v = \sqrt{\left(\frac{Es}{m} \right)}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find:

$$\sin(\theta), \cos(\theta), \tan(\theta), \sin(\varphi), \cos(\varphi), \text{ and } \tan(\varphi).$$



(3) A vector \vec{A} is given by $\vec{A} = 5\hat{i} + 4\hat{j}$. Find the following:

- (a) what is the magnitude of \vec{A} ?
- (b) what is the angle the vector makes with the x-axis?
- (c) what is the angle the vector makes with the y-axis?
- (d) Express this vector using the “hat” notation.
- (e) Express this vector using the “x-y” unit vector notation

(4) Suppose a vector \vec{B} is given by $\vec{B} = 3\hat{i} + 2\hat{j}$. Find the following:

- (a) What is $2\vec{B}$?
- (b) What is $\vec{B} + \vec{A}$?
- (c) What is $\vec{B} - \vec{A}$?
- (d) What is $\vec{B} \cdot \vec{A}$? (dot product)
- (e) What is the angle made with respect to the positive x-axis by $\vec{B} + \vec{A}$?

(5) Suppose a vector \vec{C} is given by $\vec{C} = 8\hat{i} - 9\hat{j}$. A person walks along vector \vec{A} , then vector \vec{B} , followed by vector \vec{C} . At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

(1) Which of the following equations are dimensionally correct? The following quantities have the following dimensions: {L=length, M=mass, t=time}

$$F: \left[\frac{ML}{t^2} \right]; m: [M]; a: \left[\frac{L}{t^2} \right]; t: [t]; x: [L]; E: \left[\frac{ML^2}{t^2} \right]; v: \left[\frac{L}{t} \right]; s: [L]$$

- (a) $F=ma$
- (b) $x = \frac{1}{2} at^3$
- (c) $E = \frac{1}{2} mv$
- (d) $E=\max$
- (e) $v=\sqrt{\left(\frac{Fs}{m}\right)}$

Solution:

(a) yes: $F = \left[\frac{ML}{t^2} \right]$ as dimensions. $m = [M]$, and $a = \left[\frac{L}{t^2} \right]$.

So you can see that $\left[\frac{ML}{t^2} \right] = [M] \left[\frac{L}{t^2} \right]$

(b) no: $x = [L]$; $a = \left[\frac{L}{t^2} \right]$; and $t = [t]$ so $[L] \neq [Lt]$

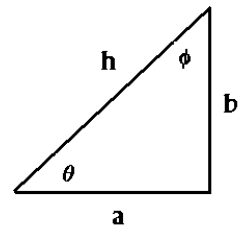
(c) no: $E = \left[\frac{ML^2}{t^2} \right]$ and $m = [M]$, $v = \left[\frac{L}{t} \right]$ so $\left[\frac{ML^2}{t^2} \right] \neq \left[\frac{ML}{t} \right]$

(d) yes: $E = \left[\frac{ML^2}{t^2} \right]$; $m = [M]$; $a = \left[\frac{L}{t^2} \right]$; $x = [L]$ so $\left[\frac{ML^2}{t^2} \right] = [M] \left[\frac{L}{t^2} \right] [L]$

(e) yes: $v = [L t^{-1}]$, $F = [M L t^{-2}]$, $s = [L]$, $m = [M]$ so $[L t^{-1}] = [L^2 t^{-2}]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find:

$\sin(\theta)$, $\cos(\theta)$, $\tan(\theta)$, $\sin(\phi)$, $\cos(\phi)$, and $\tan(\phi)$.



Solution:

(a) $h = \sqrt{a^2 + b^2}$

(b) $\sin(\theta) = \frac{b}{h}$ $\cos(\theta) = \frac{a}{h}$ $\tan(\theta) = \frac{b}{a}$

(c) $\sin(\phi) = \frac{a}{h}$ $\cos(\phi) = \frac{b}{h}$ $\tan(\phi) = \frac{a}{b}$

(3) A vector \vec{A} is given by $\vec{A} = 5\hat{i} + 4\hat{j}$. Find the following:

- (a) what is the magnitude of \vec{A} ?
- (b) what is the angle the vector makes with the x-axis?
- (c) what is the angle the vector makes with the y-axis
- (d) Express this vector using the “hat” notation.
- (e) Express this vector using the “x-y” unit vector notation

Solution:

(a) $|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} = [25 + 16]^{\frac{1}{2}} = \sqrt{41} = 6.403$

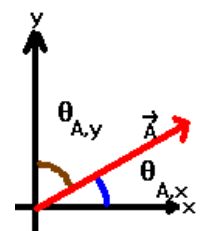
(b) $\cos(\theta_{A,x}) = \frac{\vec{A} \cdot \hat{x}}{|\vec{A}|} = \frac{(5\hat{i} + 4\hat{j}) \cdot (\hat{x} + 0\hat{y})}{\sqrt{5^2 + 4^2}} = \frac{5}{6.403} = 0.7809$ So, $\theta_{A,x} = \cos^{-1}(0.7809) = 38.66^\circ$

(c) $\cos(\theta_{A,y}) = \frac{\vec{A} \cdot \hat{y}}{|\vec{A}|} = \frac{(5\hat{i} + 4\hat{j}) \cdot (0\hat{x} + \hat{y})}{\sqrt{5^2 + 4^2}} = \frac{4}{6.403} = 0.6247$ So, $\theta_{A,y} = \cos^{-1}(0.6247) = 51.34^\circ$

My unusual notation for these angles is for clarity: these are angles with respect to the particular axis of interest. Also, note that these 2 angles add up to be 90 degrees.

(d) $\vec{A} = 5\hat{i} + 4\hat{j}$

(e) $\vec{A} = 5\hat{x} + 4\hat{y}$



(4) Suppose a vector \vec{B} is given by $\vec{B} = 3\hat{i} + 2\hat{j}$. Find the following:

(a) What is $2\vec{B}$?

(b) What is $\vec{B} + \vec{A}$?

(c) What is $\vec{B} - \vec{A}$?

(d) What is $\vec{B} \cdot \vec{A}$? (dot product)

(e) What is the angle made with respect to the positive x-axis by $\vec{B} + \vec{A}$?

$$(a) 2\vec{B} = (2 \times 3)\hat{i} + (2 \times 2)\hat{j} = 6\hat{i} + 4\hat{j}$$

$$(b) \vec{B} + \vec{A} = [3\hat{i} + 2\hat{j}] + [5\hat{i} + 4\hat{j}] = (3 + 5)\hat{i} + (2 + 4)\hat{j} = 8\hat{i} + 6\hat{j}$$

$$(c) \vec{B} - \vec{A} = [3\hat{i} + 2\hat{j}] - [5\hat{i} + 4\hat{j}] = (3 - 5)\hat{i} + (2 - 4)\hat{j} = -2\hat{i} - 2\hat{j}$$

$$(d) \vec{B} \cdot \vec{A} = [3\hat{i} + 2\hat{j}] \cdot [5\hat{i} + 4\hat{j}] = (3 \times 5) + (2 \times 4) = 15 + 8 = 23$$

$$(e) \cos(\theta_{\vec{B}+\vec{A}, \hat{x}}) = \frac{[\vec{B}+\vec{A}] \cdot \hat{i}}{|\vec{B}+\vec{A}|} = \frac{[8\hat{i}+6\hat{j}] \cdot \hat{i}}{\sqrt{8^2+6^2}} = \frac{8}{\sqrt{100}} = \frac{8}{10} = 0.8 \Rightarrow \theta = 36.87^\circ$$

Note that the power of the method in (e) which gives you a procedure to obtain the angle with respect to any unit vector. The way you can define a unit vector in an arbitrary direction is by:

$$\hat{B} \equiv \frac{\vec{B}}{|\vec{B}|}$$

For example, for this particular vector, we have:

$$\hat{B} = \frac{3\hat{i}+2\hat{j}}{\sqrt{3^2+2^2}} = \frac{3}{\sqrt{13}}\hat{i} + \frac{2}{\sqrt{13}}\hat{j} = 0.832\hat{i} + 0.555\hat{j}$$

To find the angle between the vectors A and B, you would then take the dot product between the two unit vectors. For example:

$$\begin{aligned} \cos(\theta_{\vec{A}, \vec{B}}) &= \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \hat{A} \cdot \hat{B} = \frac{[5\hat{i}+4\hat{j}]}{\sqrt{5^2+4^2}} \cdot [0.832\hat{i} + 0.555\hat{j}] = \frac{[5\hat{i}+4\hat{j}]}{\sqrt{41}} \cdot [0.832\hat{i} + 0.555\hat{j}] \\ &= \frac{4.16+2.22}{\sqrt{41}} = 0.996 \Rightarrow \theta = 4.87^\circ \end{aligned}$$

You could, of course, use the law of cosines which states:

$$\cos(\theta_{\vec{A}, \vec{B}}) = -\frac{|\vec{B}-\vec{A}|^2 - |\vec{A}|^2 - |\vec{B}|^2}{2|\vec{A}||\vec{B}|} = -\frac{[\vec{A} \cdot \vec{A} + \vec{B} \cdot \vec{B} - 2\vec{A} \cdot \vec{B}] - \vec{A} \cdot \vec{A} - \vec{B} \cdot \vec{B}}{2|\vec{A}||\vec{B}|} = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} \equiv \hat{A} \cdot \hat{B} = \frac{\vec{A} \cdot \hat{B}}{|\vec{A}|}$$

where I have used the fact that the vector pointing from \vec{A} to \vec{B} is given by $\vec{B} - \vec{A}$.

(5) Suppose a vector \vec{C} is given by $\vec{C} = 8\hat{i} - 9\hat{j}$. A person walks along vector \vec{A} , then vector \vec{B} , followed by vector \vec{C} . At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

Solution:

$$\vec{D} = \vec{A} + \vec{B} + \vec{C} = [5\hat{i} + 4\hat{j}] + [3\hat{i} + 2\hat{j}] + [8\hat{i} - 9\hat{j}] = (5 + 3 + 8)\hat{i} + (4 + 2 - 9)\hat{j} = 16\hat{i} + (-3)\hat{j}$$

$$|\vec{D}| = \sqrt{\vec{D} \cdot \vec{D}} = \sqrt{16^2 + 3^2} = \sqrt{256 + 9} = \sqrt{265} = 16.279$$

You can easily verify that $\vec{A} + \vec{B} + \vec{C} - \vec{D} = \vec{0}$.

The vector symbol over 0 is necessary because the result of adding two vectors must produce a vector.