	Physics 210: Worksheet 01	Name _____

(1) Which of the following equations are dimensionally correct? The following quantities have the following dimensions: {L=length, M=mass, t=time}

$$F:\left[\frac{ML}{t^2}\right];m:[M];a:\left[\frac{L}{t^2}\right];t:[t];x:[L];E:\left[\frac{ML^2}{t^2}\right];v:\left[\frac{L}{t}\right];s:[L]$$

(a) $F=ma$

(b) $x=\frac{1}{2}at^3$

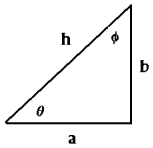
(c) $E=\frac{1}{2}mv$

(d) $E=max$

(e) $v=\sqrt{\left(\frac{Fs}{m}\right)}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find:

$$\sin(\theta),\cos(\theta),\tan(\theta),\sin(\varphi),\cos(\varphi),\text{and }\tan(\varphi).$$



(3) A vector $\vec{A}$ is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:

(a) what is the magnitude of $\vec{A}$?
(b) what is the angle the vector makes with the x-axis?
(c) what is the angle the vector makes with the y-axis
(d) Express this vector using the "hat" notation.
(e) Express this vector using the "x-y" unit vector notation

(4) Suppose a vector $\vec{B}$ is given by $\vec{B}=3\hat{i}+2\hat{j}$. Find the following:

(a)What is $2\vec{B}$?

(b) What is $\vec{B}+\vec{A}$?

(c) What is $\vec{B}-\vec{A}$?

(d) What is $\vec{B}\bullet\vec{A}$? (dot product)

(e) What is the angle made with respect to the positive x-axis by $\vec{B}+\vec{A}$?

(5) Suppose a vector $\vec{C}$ is given by $\vec{C}=8\hat{i}-9\hat{j}$. A person walks along vector $\vec{A}$, then vector $\vec{B}$, followed by vector $\vec{C}$. At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

(1) Which of the following equations are dimensionally correct? The following quantities have the following dimensions: {L=length, M=mass, t=time}

$$F:\left[\frac{ML}{t^2}\right];m:[M];a:\left[\frac{L}{t^2}\right];t:[t];x:[L];E:\left[\frac{ML^2}{t^2}\right];v:\left[\frac{L}{t}\right];s:[L]$$

(a) $F=ma$

(b) $x=\frac{1}{2}at^3$

(c) $E=\frac{1}{2}mv$

(d) $E=max$

(e) $v=\sqrt{\left(\frac{Fs}{m}\right)}$

Solution:

(a) yes: $F=\left[\frac{ML}{t^2}\right]$ as dimensions. $m=[M]$, and $a=\left[\frac{L}{t^2}\right]$.

So you can see that $\left[\frac{ML}{t^2}\right]=[M]\left[\frac{L}{t^2}\right]$

(b) no: $x=[L]; a=\left[\frac{L}{t^2}\right]$; and $t=[t]$ so $[L]\neq[Lt]$

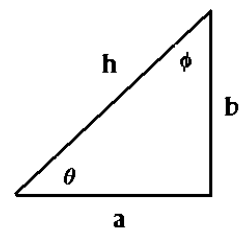
(c) no: $E=\left[\frac{ML^2}{t^2}\right]$ and $m=[M], v=\left[\frac{L}{t}\right]$ so $\left[\frac{ML^2}{t^2}\right]\neq\left[\frac{ML}{t}\right]$

(d) yes: $E=\left[\frac{ML^2}{t^2}\right]$;$m=[M]; a=\left[\frac{L}{t^2}\right]; x=[L]$ so $\left[\frac{ML^2}{t^2}\right]=[M]\left[\frac{L}{t^2}\right][L]$

(e) yes: $v=[L\,t^{-1}]$, $F=[M\,L\,t^{-2}]$, $s=[L]$, $m=[M]$ so $[L\,t^{-1}]=[L^2t^{-2}]^{1/2}$

(2) In the right triangle shown, find the following: h in terms of a and b. Then find:

$$\sin(\theta),\cos(\theta),\tan(\theta),\sin(\varphi),\cos(\varphi),\text{and }\tan(\varphi).$$



Solution:

(a) $h=\sqrt{a^2+b^2}$

(b) $\sin(\theta)=\frac{b}{h}\quad\cos(\theta)=\frac{a}{h}\quad\tan(\theta)=\frac{b}{a}$

(c) $\sin(\varphi)=\frac{a}{h}\quad\cos(\varphi)=\frac{b}{h}\quad\tan(\varphi)=\frac{a}{b}$

(3) A vector $\vec{A}$ is given by $\vec{A}=5\hat{i}+4\hat{j}$. Find the following:

(a) what is the magnitude of $\vec{A}$?
(b) what is the angle the vector makes with the x-axis?
(c) what is the angle the vector makes with the y-axis
(d) Express this vector using the "hat" notation.
(e) Express this vector using the "x-y" unit vector notation

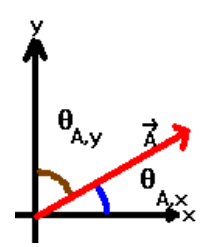
Solution:

(a) $\left|\vec{A}\right|=\sqrt{\vec{A}\bullet\vec{A}}=[25+16]^{\frac{1}{2}}=\sqrt{41}=6.403$

(b) $\cos(\theta_{A,x})=\frac{\vec{A}\bullet\hat{x}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\bullet(\hat{x}+0\hat{y})}{\sqrt{5^2+4^2}}=\frac{5}{6.403}=0.7809$ So, $\theta_{A,x}=\cos^{-1}(0.7809)=38.66^0$

(c) $\cos(\theta_{A,y})=\frac{\vec{A}\bullet\hat{y}}{|\vec{A}|}=\frac{(5\hat{i}+4\hat{j})\bullet(0\hat{x}+\hat{y})}{\sqrt{5^2+4^2}}=\frac{4}{6.403}=0.6247$ So, $\theta_{A,y}=\cos^{-1}(0.6247)=51.34^0$

My unusual notation for these angles is for clarity: these are angles with respect to the particular axis of interest. Also, note that these 2 angles add up to be 90 degrees.



(d) $\vec{A}=5\hat{i}+4\hat{j}$

(e) $\vec{A}=5\hat{x}+4\hat{y}$

(4) Suppose a vector $\vec{B}$ is given by $\vec{B} = 3\hat{i} + 2\hat{j}$. Find the following:

(a)What is $2\vec{B}$?

(b) What is $\vec{B} + \vec{A}$?

(c) What is $\vec{B} - \vec{A}$?

(d) What is $\vec{B} \bullet \vec{A}$? (dot product)

(e) What is the angle made with respect to the positive x-axis by $\vec{B} + \vec{A}$?

(a) $2\vec{B} = (2\times 3)\hat{i} + (2\times 2)\hat{j} = 6\hat{i} + 4\hat{j}$

(b) $\vec{B} + \vec{A} = \left[3\hat{i} + 2\hat{j}\right] + \left[5\hat{i} + 4\hat{j}\right] = (3+5)\hat{i} + (2+4)\hat{j} = 8\hat{i} + 6\hat{j}$

(c) $\vec{B} - \vec{A} = \left[3\hat{i} + 2\hat{j}\right] - \left[5\hat{i} + 4\hat{j}\right] = (3-5)\hat{i} + (2-4)\hat{j} = -2\hat{i} - 2\hat{j}$

(d) $\vec{B} \bullet \vec{A} = \left[3\hat{i} + 2\hat{j}\right] \bullet \left[5\hat{i} + 4\hat{j}\right] = (3\times 5) + (2\times 4) = 15 + 8 = 23$

(e) $\cos\left(\theta_{\vec{B}+\vec{A},\hat{x}}\right) = \frac{\left[\vec{B}+\vec{A}\right]}{\left|\vec{B}+\vec{A}\right|} \bullet \hat{i} = \frac{\left[8\hat{i}+6\hat{j}\right]}{\sqrt{8^2+6^2}} \bullet \hat{i} = \frac{8}{\sqrt{100}} = \frac{8}{10} = 0.8 \Rightarrow \theta = 36.87^{o}$

Note that the power of the method in (e) which gives you a procedure to obtain the angle with respect to any unit vector. The way you can define a unit vector in an arbitrary direction is by:

$$\hat{B} \equiv \frac{\vec{B}}{\left|\vec{B}\right|}.$$

For example, for this particular vector, we have:

$$\hat{B} = \frac{3\hat{i}+2\hat{j}}{\sqrt{3^2+2^2}} = \frac{3}{\sqrt{13}}\hat{i} + \frac{2}{\sqrt{13}}\hat{j} = 0.832\hat{i} + 0.555\hat{j}.$$

To find the angle between the vectors A and B, you would then take the dot product between the two unit vectors. For example:

$$\cos\left(\theta_{\vec{A},\vec{B}}\right) = \frac{\vec{A}}{\left|\vec{A}\right|} \bullet \frac{\vec{B}}{\left|\vec{B}\right|} = \hat{A} \bullet \hat{B} = \frac{\left[5\hat{i}+4\hat{j}\right]}{\sqrt{5^2+4^2}} \bullet \left[0.832\hat{i} + 0.555\hat{j}\right] = \frac{\left[5\hat{i}+4\hat{j}\right]}{\sqrt{41}} \bullet \left[0.832\hat{i} + 0.555\hat{j}\right]$$

$$= \frac{4.16+2.22}{\sqrt{41}} = 0.996 \Rightarrow \theta = 4.87^{0}$$

You could, of course, use the law of cosines which states:

$$\cos\left(\theta_{\vec{A},\vec{B}}\right) = -\frac{\left|\vec{B}-\vec{A}\right|^2 - \left|\vec{A}\right|^2 - \left|\vec{B}\right|^2}{2\left|\vec{A}\right|\left|\vec{B}\right|} = -\frac{\left[\vec{A}\bullet\vec{A}+\vec{B}\bullet\vec{B}-2\vec{A}\bullet\vec{B}\right]-\vec{A}\bullet\vec{A}-\vec{B}\bullet\vec{B}}{2\left|\vec{A}\right|\left|\vec{B}\right|} = \frac{\vec{A}\bullet\vec{B}}{\left|\vec{A}\right|\left|\vec{B}\right|} \equiv \hat{A} \bullet \hat{B} = \frac{\vec{A}\bullet\hat{B}}{\left|\vec{A}\right|}$$

where I have used the fact that the vector pointing from $\vec{A}$ to $\vec{B}$ is given by $\vec{B} - \vec{A}$.

(5) Suppose a vector $\vec{C}$ is given by $\vec{C} = 8\hat{i} - 9\hat{j}$. A person walks along vector $\vec{A}$, then vector $\vec{B}$, followed by vector $\vec{C}$. At the end of this journey, what is the displacement (vector) and the distance from the origin. You may assume all units are in m here.

Solution:

$$\vec{D} = \vec{A} + \vec{B} + \vec{C} = \left[5\hat{i} + 4\hat{j}\right] + \left[3\hat{i} + 2\hat{j}\right] + \left[8\hat{i} - 9\hat{j}\right] = (5+3+8)\hat{i} + (4+2-9)\hat{j} = 16\hat{i} + (-3)\hat{j}$$

$$\left|\vec{D}\right| = \sqrt{\vec{D} \bullet \vec{D}} = \sqrt{16^2 + 3^2} = \sqrt{256+9} = \sqrt{265} = 16.279$$

You can easily verify that $\vec{A} + \vec{B} + \vec{C} - \vec{D} = \vec{0}$.
The vector symbol over 0 is necessary because the result of adding two vectors must produce a vector.