	Physics 240: Worksheet 10	Name ___________________

(1) Suppose a mass m slides down a frictionless inclined plane (of angle θ). At the bottom of the plane, the mass encounters a flat surface with a coefficient of friction μ. How far does the mass move beyond the bottom of the plane if it falls through a vertical height y? Provide numerical answers for the case y=1m and μ=0.3.

(2) A cyclist approaches the bottom of a hill at a speed of 11 m/s. The hill is 6 m high. Ignoring friction, how fast is the cyclist moving at the top of the hill? What is the meaning of the general solution if the hill is 9 m high? :)

(3) **Using energy considerations**, what is the speed of a rock when it has fallen through a distance of 100 m if it started from rest?

(4) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal.

(5) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal and the spring is initially compressed 30 m.

Note new problem and new solution (2011).

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(1) Suppose a mass m slides down a frictionless inclined plane (of angle θ). At the bottom of the plane, the mass encounters a flat surface with a coefficient of friction μ. How far does the mass move beyond the bottom of the plane if it falls through a vertical height y? Provide numerical answers for the case y=1m and μ=0.3.

You want to apply energy conservation here. The equation is then:

$$\Delta K_{NC} = \Delta K + \Delta U$$

Now, looking at the problem from beginning to end, you see that in fact we do have the case: $\Delta K = 0$

We also have:

$$\Delta K_{NC} = W_f = -\mu mg(\Delta x)$$

We also for the gravitational potential energy have:

$$\Delta U = -mgy$$

So let's put it all together:

$$\Delta K_{nc} = \Delta K + \Delta U \Rightarrow -\mu mg(\Delta x) = -mgy \Rightarrow \Delta x = \tfrac{y}{\mu}; \Delta x = \tfrac{1}{.3} = 3.33\text{m}$$

(2) A cyclist approaches the bottom of a hill at a speed of 11 m/s. The hill is 6 m high. Ignoring friction, how fast is the cyclist moving at the top of the hill?

Solution: Total mechanical energy is conserved here. Thus, $\Delta U + \Delta K = 0$. This problem is a bit unlike other problems since the kinetic energy is not zero at any time in this problem. We need to calculate each of the relevant terms here. The terms are: $\Delta U = mgy - 0 = mgy$ and $\Delta K = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$. Let's now write the energy conservation equation:

$$\Delta K + \Delta U = 0 \Rightarrow \left[\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right] + [mgy - 0] = 0$$

No you want to solve this for the final velocity. Thus:

$$\left[\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2\right] = -mgy \Rightarrow \tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_i^2 - mgy \Rightarrow v_f^2 = v_i^2 - 2gy$$

The solution for the final velocity is then:

$$v_f = \pm\sqrt{v_i^2 - 2gy}$$

For this particular numerical example, we then get: $v_v = \pm\sqrt{v_i^2 - 2gy} = \pm\sqrt{121 - 2(9.8)6}$.

We are only asked for "how fast" in this problem so we choose the positive sign. This give us the velocity of $v_f = \sqrt{3.4} = 1.84\text{m/s}$.

(3) Using energy considerations, what is the speed of a rock when it has fallen through a distance of 100 m if it started from rest?

Solution: Total mechanical energy is conserved here. Thus, $\Delta U + \Delta K = 0$. We need to calculate each of the relevant terms here. $\Delta U = -mgy - 0 = -mgy$ and $\Delta K = \tfrac{1}{2}mv^2 - 0 = \tfrac{1}{2}mv^2$. Thus, $-mgy + \tfrac{1}{2}mv^2 = 0 \Rightarrow v = \pm\sqrt{2gy}$. The – solution is the physical solution here. Putting in the values, we find:

$$v = -\sqrt{2(9.8)100} = -\sqrt{1960} = -44.3\text{m/s}.$$

(4) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal.

This problem conserves total mechanical energy. Thus: $\Delta U + \Delta K = 0$. We need to calculate each of the relevant terms. $\Delta U = \frac{1}{2}k\left[x_f^2 - x_i^2\right]$ and $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}mv^2 = -\frac{1}{2}mv^2$. Let's put everything together. I'm going to assume the spring is initially at x_i=0.

$\Delta U + \Delta K = 0 \Rightarrow \Delta U = \frac{1}{2}k\left[x_f^2 - x_i^2\right] - \frac{1}{2}mv^2 = 0 \Rightarrow x_f = \pm\sqrt{\frac{m}{k}}v$

The final position of the mass is then:

$$x_f = \pm\sqrt{\frac{m}{k}}v$$

But in this problem, we're asked "how much will the spring compress" so we're looking for Δx. The symbolic problem solution is then:

$$\Delta x = \pm\sqrt{\frac{m}{k}}v$$

If the mass is initially traveling in the +x direction, then the physical solution is the positive

$\frac{1}{2}kx^2 - \frac{1}{2}mv^2 = 0$. Solve this for x: $x = \pm\sqrt{\frac{m}{k}}v$. I'll assume the mass is moving in the +x direction initially. Then the physical solution is +. Let's put in the values to find then $\Delta x = \sqrt{\frac{100\text{kg}}{5\frac{\text{N}}{\text{m}}}}15\frac{\text{m}}{\text{s}} = \sqrt{\frac{100\text{kg}}{5\frac{\text{kgm/s}^2}{\text{m}}}}15\frac{\text{m}}{\text{s}} = \left[\sqrt{20\text{s}^2}\right]15\frac{\text{m}}{\text{s}} = 67.1\text{m}$

(5) Suppose a 100 kg mass is traveling with a velocity of 15 m/s. If the mass strikes a spring with a spring constant k=5 N/m, how much will the spring compress before the mass stops. This system is horizontal and the spring is initially compressed 30 m.

The spring was already compressed through a distance or 30 m and the mass causes it to compress an additional x m. Thus the final potential energy of the spring was:

$U_f = \frac{1}{2}k\left(x+30\right)^2$. The initial potential energy of the spring (before the mass struck it was:

$U_i = \frac{1}{2}k\left(30\right)^2$. Thus we obtain the change in potential energy as:

$\Delta U = U_f - U_i = \frac{1}{2}k\left[\left(x+30\right)^2 - 30^2\right] = \frac{1}{2}k\left[x^2 + 60x\right]$

All of this change comes from the change in kinetic energy of the mass:

$\Delta K_c = K_f - K_i = -\frac{1}{2}m\left(v^2\right) = -\frac{1}{2}100\left(15\right)^2 = -11250\text{J}$

So we actually do need to solve the equation:

$0 = \Delta K_c + \Delta U = -11250 + 2.5x^2 + 75x$

$\Rightarrow x^2 + 30x - 4500 = 0 \Rightarrow x = \frac{-30\pm\sqrt{30^2-4(-4500)}}{2} = \frac{-30\pm137.4}{2}$

Choose the positive solution here:

X=53.7m. The total compression of the spring would be 93.7m.

All in all, you really need to be careful about superimposition of energies, however. In particular, from classical physics relative velocities (in one dimension) will add but the result is that the kinetic energy is not what is given by K1+K2. For example: if a 1kg mass is traveling with a speed of 2 m/s and a 1 kg mass is traveling at a speed of 4 m/s, the total kinetic energy of the second mass is not 4 J; instead it is 8 J.