

A mass  $m_1$  collides and sticks to a mass  $m_2$ . After the collision, the mass combination is seen to move with a speed  $v$ .

**(a)** If mass  $m_2$  was at rest before the collision, how fast was mass  $m_1$  moving before the collision in terms of  $m_1$ ,  $m_2$  and  $v$ ?

**(b)** Suppose the mass combination encounters a frictionless inclined plane with a moderate angle of inclination. How far above the base of the plane will the mass combination be at the instant it stops in terms of  $v$  and  $g$ ?

**(c)** If  $m_1$  acquired its original momentum from the decompression of a spring of spring constant  $k$ , how much was the spring compressed in terms of  $k$ ,  $m_1$ ,  $m_2$  and  $v$ ?

**(d)** Suppose a force  $|\vec{F}|=4c$  from  $t=0$  to  $t=4s$  was applied to the mass. What was the change in velocity of the mass if the mass was 8 kg and  $c=1$  N ?

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**(a)** If mass  $m_2$  was at rest before the collision, how fast was mass  $m_1$  moving before the collision in terms of  $m_1$ ,  $m_2$  and  $v$ ?

Assuming that the collision is totally inelastic, we have:

$$\Delta \vec{P} = \vec{0} \Rightarrow m_1 v_{1,b} + m_2 v_{2,b} = (m_1 + m_2) v_A : v_{2,b} = 0 \Rightarrow v_{1,b} = \frac{m_1 + m_2}{m_1} v$$

**(b)** What fraction of the initial kinetic energy was lost in this collision in terms of  $m_1$ ,  $m_2$  and  $v$ ?

For a totally inelastic collision,

$$\begin{aligned} \text{fraction lost} &= 1 - \frac{K_f}{K_i} \\ &\Rightarrow 1 - \frac{\frac{1}{2}(m_1 + m_2)v^2}{\frac{1}{2}m_1 v_{1,b}^2} = 1 - \frac{(m_1 + m_2)v^2}{m_1 \left(\frac{m_1 + m_2}{m_1}\right)^2 v^2} = 1 - \frac{m_1}{m_1 + m_2} = \frac{m_1 + m_2}{m_1 + m_2} - \frac{m_1}{m_1 + m_2} \\ &\Rightarrow \text{fraction lost} = \frac{m_2}{m_1 + m_2} : \% \text{ lost} = 100 \times \frac{m_2}{m_1 + m_2} \end{aligned}$$

In particular, note that if  $m_2 = 0$  then no kinetic energy is lost.

**(c)** If  $m_1$  acquired its original momentum from the decompression of a spring of spring constant  $k$ , how much was the spring compressed in terms of  $k$ ,  $m_1$ ,  $m_2$  and  $v$ ?

$$\begin{aligned} \Delta K_{NC} &= \Delta K_C + \Delta U \\ \Delta K_{NC} = 0 : \Delta K_C &= \frac{1}{2} m_1 v_{1,b}^2 : \Delta U = \Delta U_s = -\frac{1}{2} k x^2 \Rightarrow 0 = \frac{1}{2} m_1 v_{1,b}^2 - \frac{1}{2} k x^2 \Rightarrow x = \sqrt{\frac{m_1}{k}} v_{1,b} \\ x &= \sqrt{\frac{m_1}{k}} \frac{(m_1 + m_2)}{m_1} v = \sqrt{\frac{(m_1 + m_2)^2}{k m_1}} v \end{aligned}$$

**(d)** Suppose a force  $|\vec{F}| = 4c$  from  $t = 0$  to  $t = 4s$  was applied to the mass. What was the change in velocity of the mass if the mass was 8 kg and  $c = 1$  N?

$$\begin{aligned} \vec{J} &= \vec{F} \Delta t = \Delta \vec{p} = m \Delta \vec{v} \Rightarrow \Delta \vec{v} = \frac{\vec{F} \Delta t}{m} \\ \text{One dimension} &\Rightarrow \Delta v = \frac{(4)(1)(4)}{8} = \frac{16}{8} = 2 \frac{m}{s} \end{aligned}$$