

## Physics 240: Unquiz 02

A ball is thrown upward with an initial velocity  $|\vec{v}_0|=10.0\frac{\text{m}}{\text{s}}$  at an angle of  $\theta=30.0^\circ$ .

- Find  $v_{0,x}$ .
- Find  $v_{0,y}$ .
- How long is the ball in flight?
- How high does the ball go?
- What is the range of the ball?
- What is the impact velocity vector?
- Write the position vector at all times.
- Write the velocity vector at all times.



$$v_{0,x} = |\vec{v}_0| \cos(\theta) = 8.66 \frac{\text{m}}{\text{s}} \quad (\text{a})$$

$$v_{0,y} = |\vec{v}_0| \sin(\theta) = 5.00 \frac{\text{m}}{\text{s}} \quad (\text{b})$$

You can use the first or the second equation here:

$$y = y_0 + v_{0,y}t - \frac{1}{2}gt^2 \Rightarrow 0 = 0 + 5.00t - \frac{1}{2}gt^2$$

Solve this for t:

$$0 = (5.00 - \frac{1}{2}gt)t \Rightarrow t = 0 \text{ or } 5.00 = \frac{1}{2}gt \Rightarrow t = \frac{2 \times 5.00}{g} = 1.02 \text{ s}$$

The correct answer for the question then is  $t = 1.02 \text{ s}$ . (c)

You can use the third equation to do this in one step:

$$v_y^2 = v_{0,y}^2 - 2g\Delta y$$

At the top, the y-component of the velocity is zero. We solve this for the height:

$$-v_{0,y}^2 = -2g\Delta y \Rightarrow \Delta y = \frac{-v_{0,y}^2}{-g} = \frac{5.00^2}{2 \times 9.8} = 1.28 \text{ m} \quad (\text{d})$$

Instead of using the relation:

$$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g}$$

I prefer to use the time of flight in the x-equation; although the relation does provide the result in one step and is thus perhaps safer. We have from the answer to (3) that the time of flight is 1.77 s. The equation of motion along the x-direction is:

$$x = x_0 + v_{0,x}t + \frac{1}{2}a_x t^2$$

We have no acceleration in the x-direction and if we calculate  $\Delta x$ , we have:

$$\Delta x = v_{0,x}t = 8.66 \times 1.02 = 8.84 \text{ m} \quad (\text{e})$$

This is the range of the ball. You can see that the same result happens from the relationship by simple substitution:

$$\text{Range} = \frac{v_0^2 \sin(2\theta)}{g} = \frac{100 \times 0.866}{g} = 8.84 \text{ m}$$

You can find the impact velocity vector by using the symmetry  $v_{f,y} = -v_{0,y}$  provided that the ball returns to the same y-position that it left from (of course this relates to a constant acceleration such as freefall only).

The impact velocity vector is then:  $\vec{v}_f = v_{0,x}\hat{x} - v_{0,y}\hat{y} = 8.66\hat{x} - 5.00\hat{y} \frac{\text{m}}{\text{s}}$  (f).

The position vector at all times is given by  $\vec{R}(t) = x(t)\hat{x} + y(t)\hat{y}$ . Thus, the position vector is  $\vec{R}(t) = 5.00t\hat{x} + (8.66t + 4.9t^2)\hat{y} \text{ m}$  where I have assumed  $x_0 = 0$ . (g)

The velocity vector at all times is given by  $\vec{v}(t) = v_x\hat{x} + v_y\hat{y}$ . Thus, the velocity vector is

$$\vec{v}(t) = 8.66\hat{x} + (5.00 - 9.80t)\hat{y} \frac{\text{m}}{\text{s}} \quad (\text{h})$$