Friday, October 24, 2014	Physics 210: UnQuiz06	Name:_____

A spring has a spring constant of 90 N/m. A mass m=10 kg is attached to the spring. Find the period of oscillations.

If the mass is initially pulled to a distance of +0.4 m from the equilibrium position, and released from rest, find the total energy of the system.

Find the maximum velocity of the system.

Find the equations of motion.

Suppose the mass were passing through the equilibrium position at t=0 with a velocity of +1.2 m/s. Find the equations of motion.

How fast was the mass moving when x=+A/2?

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$$\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{90\,N/m}{10\,kg}}=\sqrt{9\,rad^2/s^2}=3\,rad/s:\omega=2\pi f\Rightarrow f=\frac{\omega}{2\pi}=\frac{3}{2\pi}=0.48\,Hz:T=\frac{1}{f}=2.1\,s$$

If the mass is initially pulled to a distance of +0.4 m from the equilibrium position, and released from rest, find the total energy of the system.

$$E=\frac{1}{2}kA^2=\frac{1}{2}x90\,N/m\,x(0.4\,m)^2=7.2\,J$$

Find the maximum velocity of the system.

$$E=K+U;\text{ if } U=0\Rightarrow E=K_{max}=\frac{1}{2}mv_{max}^2\Rightarrow v_{max}=\sqrt{2\frac{E}{m}}=\sqrt{2x7.2\,J/10\,kg}=1.2\,m/s$$

Find the equations of motion.
Here, a phase of zero will provide the correct solution. The phase is given by:

$$x(t)=A\cos(\omega t+\phi);v(t)=-\omega A\sin(\omega t+\phi);a(t)=-\omega^2x(t)$$

$$\tan\phi=\frac{v(0)}{x(0)}=0\Rightarrow\phi=\tan^{-1}(0)=0\text{ or }\pi;x(0)>0\Rightarrow\phi=0$$

$$x(t)=0.4\cos(3t):v(t)=-1.2\sin(3t):a(t)=-3.6\cos(3t)$$

Suppose the mass were passing through the equilibrium position at t=0 with a velocity of +1.2 m/s. Find the equations of motion.

$$\tan(\phi)=\frac{v(0)}{x(0)}=\infty\Rightarrow\phi=\frac{+\pi}{2}\text{ or }-\frac{\pi}{2};v(0)>0\Rightarrow\phi=\frac{+\pi}{2}$$

$$\frac{1}{2}kA^2=\frac{1}{2}mv_{max}^2\Rightarrow A=\sqrt{\frac{m}{k}}v_{max}=\sqrt{\frac{10}{90}}x1.2=0.4\,m$$

$$x(t)=0.4\cos(0.3t+\frac{\pi}{2});v(t)=-1.2\sin(0.3t+\frac{\pi}{2});a(t)=-3.6\cos(0.3t+\frac{\pi}{2})$$

How fast was the mass moving when x=+A/2?

$$E=K+U\Rightarrow 7.2\,J=\frac{1}{2}mv^2+\frac{1}{2}k(0.2)^2\Rightarrow 7.2\,J=5v^2+1.8J\Rightarrow v=\sqrt{\frac{7.2-1.8}{5}}=1.04\,m/s$$

Note that the velocity at this point is -1.04 m/s.