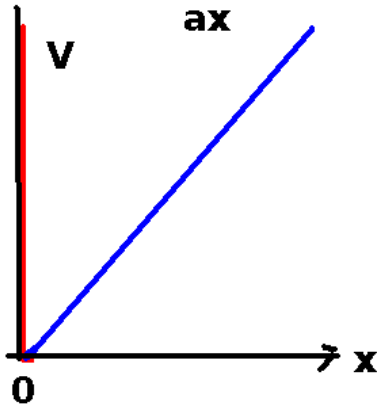


We consider an infinite square well with a sloped potential:
a semi-qualitative abbreviated approach to the problem.



The 1DTISWE:

$$-\frac{\hbar^2}{2m} \frac{d^2\phi}{dx^2} + ax\phi = E\phi$$

Change variables:

$$u \equiv \left(\frac{2m}{\hbar^2} a\right)^{1/3} \left(x - \frac{E}{a}\right)$$

Then the SWE becomes:

$$\frac{d^2\phi}{dx^2} - u\phi = 0$$

The solutions to this equation are Airy functions $Ai(x)$ and $Bi(x)$.

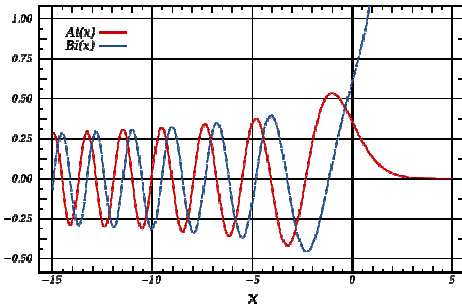
We require here that at $x=0$, the wavefunction vanishes.

At infinity the wave function also must vanish. This allows us to discard $Bi(x)$.

Note that

$$u(0) = -\left(\frac{2m}{\hbar^2} a\right)^{1/3} \frac{E}{a}$$

So we'd only retain the Ai .



This is from Wikipedia and shows the two functions.
We'll only use the red solutions here.

The zeros of the Airy functions give the energies.

$$Ai(u')=0$$

Look at dlmf.nist.gov

The first zero is when $u'=-2.3381$ which gives the ground energy:

$$\left(\frac{2m}{\hbar^2} a\right)^{1/3} \frac{E_1}{a} = 2.3381 \Rightarrow \frac{E_1}{a} = 2.3381 \left(\frac{\hbar^2}{2ma}\right)^{1/3} \Rightarrow E_1 = 2.3381 \left(\frac{a^2 \hbar^2}{2m}\right)^{1/3}$$

The second energy would be:

$u'=-4.0879$ which gives:

$$\left(\frac{2m}{\hbar^2} a\right)^{1/3} \frac{E_2}{a} = 4.0879 \Rightarrow \frac{E_2}{a} = 4.0879 \left(\frac{\hbar^2}{2ma}\right)^{1/3} \Rightarrow E_2 = 4.0879 \left(\frac{a^2 \hbar^2}{2m}\right)^{1/3}$$

etc.

Some points to note is that as x increases, fewer and fewer squiggles are present.

Mostly the particle is going to try to bunch up in the lower potential regions.

In a slightly different problem, without the wall at zero, you would need to worry about symmetries and solution numbering may be different.