

De Broglie Waves

(1924 – 1927)

Awareness of Einstein's work showing mass-energy equivalence

1924 Thesis: at the Faculty of Sciences at Paris University

Recherches sur la Théorie des Quanta

Basically he postulated that matter must have wave-like properties:

$$\lambda = \frac{h}{p}$$

This may be how he arrived at this hypothesis:

For a photon:

$$E = pc$$

which was known classically. The energy was shown to be given by:

$$E = hf$$

Thus if the two expressions are equated, we find:

$$hf = pc \Rightarrow h = p \frac{c}{f} = p\lambda \Rightarrow \lambda = \frac{h}{p}$$

De Broglie's extension was to suggest that this simple relationship held for all particles, not just photons. This is now called the De Broglie wavelength of a particle.

example: What is the De Broglie wavelength of a tennis ball of mass 70 g traveling at 25 m/s and an electron of energy 50 eV.

For the tennis ball:

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34} \text{ JS}}{(0.07 \text{ kg})(25 \text{ m/s})} = 3.8 \times 10^{-34} \text{ m}$$

For the electron, first recognize that it is produced by accelerating an electron through a potential difference of 50 V. For the non-relativistic electron we then have:

$$E = \frac{p^2}{2m} \Rightarrow p = \sqrt{2mE} = \frac{\sqrt{2(mc^2)E}}{c} \Rightarrow \lambda = \frac{hc}{\sqrt{2(mc^2)E}} = \frac{1240 \text{ eVnm}}{\sqrt{2(0.511 \times 10^6 \text{ eV})(50 \text{ eV})}} = 0.17 \text{ nm}$$

This is a very important result since it says that acceleration of electrons through only a moderate potential difference will produce a wavelength on the order of crystalline structure size. As we will see, this accidental correlation becomes significant.

Let's look at Bohr's quantization condition in the light of the De Broglie wavelength.

Suppose we have standing waves on the Bohr orbitals. In order for a standing wave to exist, we have:

$$n\lambda = 2\pi r$$

where r is the orbital radius. We now use the De Broglie relation here:

$$n \left(\frac{h}{p} \right) = 2\pi r$$

The angular momentum of the electron is given by $L = rp$. We thus have:

$$n \left(\frac{h}{2\pi} \right) = rp = L \Rightarrow L = n\hbar$$

This means that Bohr's assumption follows as a very natural consequence of the condition for a standing wave to exist from De Broglie's wavelength. This is wonderful, but your author says that proof of the wave nature of electrons is still lacking.

A rather simple experiment of Davisson Germer (1927) at Bell Labs turns out to have provided the proof of the wave nature of electrons.

I'll leave it up to you to read the text here regarding the background. Suffice it to say that not every accident is bad.

In any event, the pair observed sharp peaks at an angle of about 50 degrees in the scattered electrons from their experiment. This was only observed after they had cleaned their crystal and apparently produced large single crystal domains in their nickel. Ultimately, this was correctly interpreted as evidence for the wave nature of the electron.

In lab, we will also scatter electrons from a target and directly observe this wave nature associated with the electron.