The charge to mass ratio of the electron

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In this experiment, you are going to duplicate a classical experiment of modern physics. Late in the 1800's JJ Thompson was able to provide a measurement of the e/m ratio of the electron by deflecting an electron beam through a magnetic field. Your experiment is basically the same idea as this first experiment but it has the advantage of 100 years of evolution to become a relatively simple experiment to perform. In the late 1800's however, this experiment was very much at the cutting edge of technology.

Here is how the experiment is supposed to work. A heater voltage is applied to a filament. The filament heats up and thus makes electrons relatively easy to release if a potential difference is applied in this region. The liberated electrons are accelerated through this potential difference to a velocity, v. We can find this velocity by equating the work to accelerate an electron through a potential difference to the resulting change in kinetic energy (assuming that no work is done to remove the electrons from the filament):

$$eV = \tfrac{1}{2}mu^2 \Rightarrow u = \sqrt{\frac{2eV}{m}}$$

Here, e is the charge on the electron, V is the potential difference, m is the mass of the electron and v is the velocity of the electron.

Now, suppose that these electrons are subjected to a magnetic field which is at right angles to the velocity. Then, you know the electron will experience a force given by

$$\vec{F} = e\vec{u}\times\vec{B} \Rightarrow \left|\vec{F}\right| = e\vec{u}B$$

Since the force is always at right angles to the velocity, circular motion will result. You may recall from general physics that the acceleration for uniform circular motion is given by:

$$a_c = \frac{u^2}{R}$$

with R being the radius of orbit. Since the Lorentz force produces this acceleration, we can equate the two to find another expression for the velocity:

$$euB = m\tfrac{u^2}{R} \Rightarrow u = \tfrac{e}{m}BR$$

But, we know what the velocity of the electron is from the first equation. I will use that to obtain a value for the ratio of the charge to mass for the electron:

$$\sqrt{\tfrac{2eV}{m}} = \tfrac{e}{m}BR \Rightarrow \tfrac{2eV}{m} = \left(\tfrac{e}{m}\right)^2 B^2R^2 \Rightarrow \tfrac{e}{m} = \tfrac{2V}{B^2R^2}$$

So, if we can measure V and B and the radius of orbit, we have all the information necessary to measure (e/m)! In the present lab, our magnetic field is produced by Helmholtz coils which are much like a solenoid, only different. An important property of Helmholtz coils, however, is that they produce a uniform magnetic field in their center. It can be shown that the value of this magnetic field is approximately given by

$$B = \frac{(N\mu_0)I}{(5/4)^{3/2}a}$$

where N is the number of turns on a coil (here, N=130), I is the current through the coil, a is the radius of the coils and μ_0 is the permeability of free space ($4\pi x10^{-7}$ in SI units).

Calculation of the magnetic field for Helmholtz coils

From the law of Biot-Savert:

$$d\vec{B} = \frac{\mu_o I d\vec{l} x \vec{r}_{ip}}{4\pi \vec{r}_{ip}^3}$$

We use cylindrical coordinates:

$$d\vec{l} = ad\theta\hat{\theta} = ad\theta\left[-\sin\theta\hat{x} + \cos\theta\hat{y}\right]$$

$$\vec{r}_{ip} = -a\cos\theta\hat{x} - a\sin\theta\hat{y} + z_p\hat{z}$$

We only need the z-component of the cross product because after integration, along the symmetry axis, the x and y components will vanish. Thus:

$$\left[d\vec{l} \times \vec{r}_{ip}\right]_z = a^2 d\theta \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ -\sin\theta & \cos\theta & 0 \\ -\cos\theta & -\sin\theta & \frac{z_p}{a} \end{vmatrix} \bullet \hat{z} = a^2 d\theta\left[\sin^2\theta + \cos^2\theta\right] = a^2 d\theta$$

So the z-component of the magnetic field is:

$$d\vec{B} \bullet \hat{z} = \frac{\mu_o I a^2 d\theta}{4\pi a^3\left[1^2 + \left(\frac{z_p}{a}\right)^2\right]^{3/2}} = \frac{\mu_o I d\theta}{4\pi a\left[1^2 + \left(\frac{z_p}{a}\right)^2\right]^{3/2}}$$

We now find the vector magnetic field:

$$\vec{B} = \int d\vec{B} = \int_{\theta=0}^{\theta=2\pi} \frac{\mu_o I d\theta}{4\pi a\left[1^2 + \left(\frac{z_p}{a}\right)^2\right]^{3/2}}\hat{z} = \frac{\mu_o I}{2a\left[1^2 + \left(\frac{z_p}{a}\right)^2\right]^{3/2}}\hat{z}$$

A second Helmholtz coil is at a distance a from the first helmholtz coil and the position of interest is ½ way between the two coils. This gives us then:

$$\vec{B} = 2x\frac{\mu_o I}{2a\left[1^2 + \left(\frac{1}{2}\right)^2\right]^{3/2}}\hat{z} = \frac{\mu_o I}{a\left[\frac{5}{4}\right]^{3/2}}\hat{z}$$

Each coil has a total of N turns. This then gives:

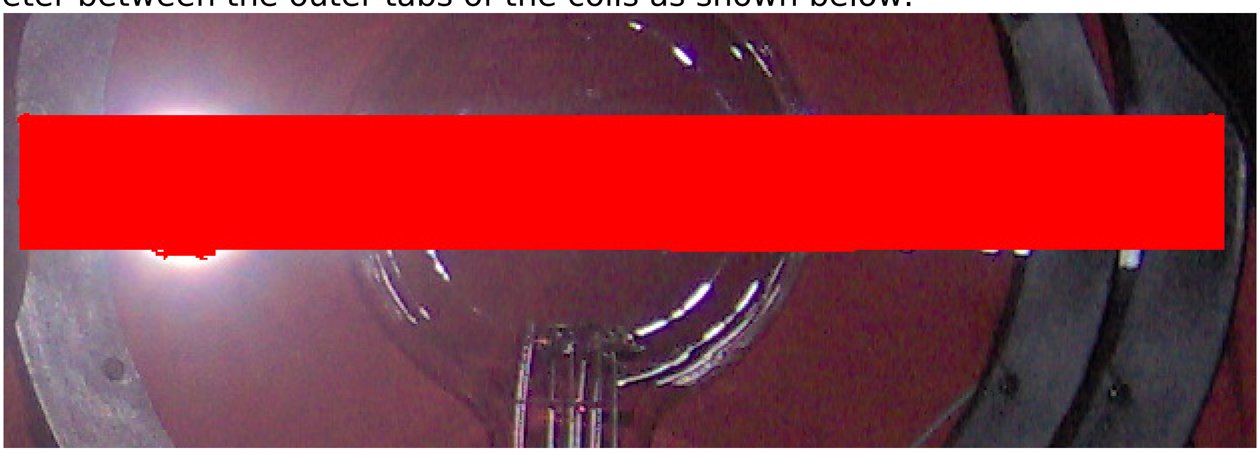
$$\vec{B} = \frac{\mu_o N I}{a\left[\frac{5}{4}\right]^{3/2}}\hat{z}$$

If we use this magnetic field in our expression for (e/m) we find that the ratio is given by

$$\frac{e}{m} = \frac{2V}{B^2R^2} = \frac{2V}{R^2}\frac{1}{\left[\dfrac{\mu_0 NI}{a\left[\frac{5}{4}\right]^{3/2}}\right]^2} = \frac{2Va^2\left[\frac{5}{4}\right]^3}{R^2(\mu_0 NI)^2}$$

So the procedure is relatively clear now ... measure and record a. Vary (and record) V and I in such a way that the radius of orbit of the electron beam is a nice circle which has the ruler mounted on the back of the apparatus centered across a diameter. (This is why you need to correctly vary both V and I keeping R constant).

Your circle should be centered on about +- 4 cm. It won't be. So measure both sides and average the two to get your radius. To measure a, I suggest you measure the diameter between the outer tabs of the coils as shown below.



That measurement is a fairly large source of error and we can not do much better than this without taking the coils apart. See if you can agree with me with 30.9 cm here.

The procedure now is to increase your voltage from 0 until you are able to get your circle **and its reflection** to center on the 4 cm marks (or what you have chosen). This last step is very important ... a small amount of parallax will lead to rather large experimental errors here. You will need to vary the current in order to get a perfect fit. Next, increase your voltage from this lowest voltage up to about 350 volts in 20 volt steps. At each 10 volt change, you will change the current in such a way so as to keep the circle centered on the 4 cm marks (or what you have chosen). Record the voltage and current needed at each point. Upon completion, turn your voltage and current down to low levels.

Analysis

If you look back at your expression for the (e/m) ratio, it was:

$$\frac{e}{m} = \frac{2Va^2\left[\frac{5}{4}\right]^3}{R^2(\mu_0 NI)^2}$$

Let's define a constant C which is given by:

$$C \equiv \frac{R^2(\mu_0 N)^2}{2a^2\left[\frac{5}{4}\right]^3} .$$

The above expression reduces to:

$$\frac{e}{m} = \frac{1}{C}\frac{V}{I^2} \Rightarrow I^2 C \frac{e}{m} = V .$$

What you want to do is to plot the variable as:

$$x \equiv I^2 \Rightarrow V = \text{slope} \times x$$

Where the slope is given by:

$$\text{slope} = C\frac{e}{m}$$

At this point, the charge to mass ratio is then given by:

$$\frac{e}{m} = \frac{\text{slope}}{C}$$

Here, R is 4 cm (0.04 m or what ever you chose, averaged, of course. **Make sure you measure both sides, also you might stick your head into the apparatus is your eyesight is as bad as mine is.** N is 130, and a is the radius of the Helmholtz coils. Calculate the value of this constant (You can do this by entering the circle radius on your spreadsheet). Now, in your excel spreadsheet, enter your values of I and V in the column provided. The spreadsheet will then produce a column of current^2. This will be plotted as a function of the accelerating potential V.

If everything worked perfectly, this plot would be a straight line with a slope equal to (e/m). The spreadsheet will make a linear fit to your plot and provide the slope of this line. Enter the slope in your spreadsheet in order to determine the charge to mass ratio of the electron. The accepted value for the (e/m) ratio is (e/m)=-1.76x10^{11} C/kg. Compare your slope (you can neglect the negative sign in the accepted value) to the accepted value by use of the percentage error. Don't be surprised if you are off somewhat ... after all, the velocity of the electrons must be in error since the only way the gas in the tube can glow is if it is excited by the electrons, and this very action removes kinetic energy from the electrons. You should note that a correction for the Earth's magnetic field is not all that large in this experiment. You can confirm this by looking at the beam deflection when the external magnetic field is completely off.