Simple Harmonic Oscillation

Revised Fall 2012

In this lab, you will experiment with two simple harmonic oscillators, namely the simple pendulum and the spring – mass system. Let's go through the analysis of each of these systems.

From class notes, you know that if a force **F** is restoring (meaning that it points towards an equilibrium position or in the opposite direction of the displacement) and is linear in the displacement variable, then the motion is described by:

$$x(t) = A\cos(\omega t + \phi),$$
$$v(t) = -\omega A\sin(\omega t + \phi),$$
$$a(t) = -\omega^2 x(t).$$

These results come from application of Newton's law to the following situation:

(a) The force must be linear and restoring in a generalized displacement variable, Z. This means it is of the form:

$$F = -(\text{constant}) \times Z^1$$

The negative sign says restoring while Z^1 means that the function is linear in the displacement variable.

(b) By Newton's law we have then $F = ma_z$

Where here, a_z is generalized acceleration of the displacement variable Z and m is a generalized inertial variable (for example, moment of inertia).
Equating these two forces provides the result:

$$-(\text{constant}) \times z = ma_z$$

We put this into standard form to read:

$$a_z + \tfrac{\text{constant}}{m} z = 0$$

Once the equation is cast into this form, the frequency of oscillation about the equilibrium position is given by:

$$\omega = \sqrt{\tfrac{\text{constant}}{m}}$$

In these equations, Z represents a generalized position, v represents a generalized velocity and a represents a generalized acceleration. We also showed in class that **for the spring mass system**, Z can represent the correct position as a function of time if

$$\omega = \sqrt{\frac{k}{m}}$$

where k is the spring constant and m is the mass attached to the spring. For the simple pendulum, we also showed that the condition for the angular frequency must be given by

$$\omega = \sqrt{\frac{g}{L}}$$

where g is the acceleration due to gravity and L is the length of the simple pendulum.

In practical terms, however, it is not so easy to measure ω. It is much easier to measure either T (period) or f (frequency). These quantities are all related to ω quite simply:

$$\omega = 2\pi f \text{ , } \omega = 2\pi / T \text{ and } T = 1/f.$$

This means that the period of the spring-mass system should be given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

and the period of the simple pendulum should be given by

$$T = 2\pi\sqrt{\frac{L}{g}} \text{ .}$$

Procedure

Part 1: The period of a spring-mass system.

A note on counting: say "zero" at the instant you release the mass.

a:

You will need to measure the spring constant of the spring provided to you first. Do this starting with 5 grams mass on the spring and increasing the mass (and thus the force) up to a maximum of about 45 grams in 5 gram increments. This is almost identical to a measurement you made in a previous lab (see lab 5) so this should go very quickly for you. There is a section of the spreadsheet which will help here. The slope of a graph of force vs. displacement will be your spring constant.

b:

Now that you have measured your spring constant, you are ready to measure the frequency of oscillation for your system.

Place 105g on your spring (the spring is 632-050). Time 20 oscillations and find the time per oscillation.
Be aware that at the moment you release the mass and start the stop watch, you should say "zero", not "one".

Repeat this for 125g, 155g, 175g, and 205g masses.

For the spring mass system, we have:

$$\omega = \sqrt{\frac{k}{m}} = \frac{2\pi}{T} \Rightarrow \frac{T}{2\pi} = \sqrt{\frac{m}{k}} \Rightarrow T^2 = \frac{4\pi^2}{k} m$$

If our development is correct for the spring-mass system, then you should be able to show by plotting a graph with T^2 on the y-axis and m on the x-axis that the slope of a line passing through these three points is equal to $\frac{4\pi^2}{k}$. Compare the spring constant obtained this way to that you measured earlier by use of the %error.

Part 2: The period of a simple pendulum.

a:

Choose one of the small steel balls provided. Connect enough fishing line to this mass so that you can vary your length between 0.5 m and 7 m. In the first part of this portion of the lab, you will verify that the period of a simple pendulum varies as $\sqrt{L}$ where L is the length of the pendulum.

Measure the length from your holder to the pendulum until you have a length of 0.5 m. Time 20 oscillations of your pendulum and find the time per oscillation. Repeat this for lengths of about [1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0]m keeping the mass constant. As the string gets longer, you will probably need to move out to the Derby Center steps.

For the simple pendulum: $T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow T^2 = \frac{4\pi^2}{g} L$. If our development of the simple pendulum is correct, then you should be able to show by plotting a graph with T^2 on the y axis and L on the x axis that the slope of the line fit to these data points has a slope given by $\frac{4\pi^2}{g}$. Compare the slope of your graph to this theoretical value by use of the %error.

b:

Now, select two other masses. Attach these to your pendulum and set the length to the center of mass of the larger masses to be 1.0 m. For each of these masses, leave the length the same and time 20 complete oscillations All together, you should have made three measurements at the length of 1.0 m and you should weigh each of the masses that you have used. If our formulation of the simple pendulum is correct, then the period should be independent of the mass of the pendulum. You can show this by plotting a graph with T on the y axis and m (in kg) on the x axis. A line passing through these points should have a slope of almost 0 (zero) meaning that the period of the simple pendulum is not dependant upon the mass of the simple pendulum. You probably won't get exactly zero here. You will want to think of some reasons that this is the case. Some hints about this: (1) the string actually does stretch and (2) you might not have exactly 1 m of string length to the center of mass of the three masses. This can be corrected by slightly increasing the length of the string as you put additional masses on the string. However, if you've done your experiment correctly here, you should be able to interpret the slope of your graph as showing almost no dependence upon mass for the period of the simple pendulum. Compare, for example, this slope to the previous slope.

Part 3: Slinky investigations

In the last part, I want you to measure the period of the slinky as a function of the length of the slinky. You will probably need to start with 30 turns hanging, and then increase in steps of 10 until you have about 80 turns. I have a separate spreadsheet for this part of the lab which will plot the period vs. length. If your results are like mine, the result will be a linear fit.

Write-up

In addition to the three graphs discussed above, your write up should discuss the two problems that are under consideration. From your data, you should be able to determine if the formulation is in agreement with reality. You should discuss possible sources of deviation in your results. You should also take this opportunity to make sure you completely understand the connection between frequency (f), period (T) and angular frequency (ω).