Energy and power

Energy, work and power in physics are defined. Work is a force exerted (on matter) which is accompanied by a displacement along the direction of the force. By the work-energy theorem, work produces a change in energy. Power is the rate of doing work.

One notational comment: in the following notes, s represents the displacement. I am defining the work done on a system by an external entity which is exerting a force $\vec{F}$ on the system while the system is undergoing a displacement $\Delta\vec{s}$ in the direction of the applied force as positive. In the analysis below, I will try to be painfully clear about the various signs involved.

When you walk up stairs, you do work against the gravitational force by exerting a force on a step. The force (Note: it is in the negative y direction) which you exert to increase your height is given by:

$$\vec{F}_{\text{on step}} = m\vec{g} = -mg\hat{y}$$

where m is your mass and $g = +9.8\frac{m}{s^2}$ which is the **magnitude** of the **vector** acceleration due to gravity. This required force is slightly higher than this in order to ascend (or slightly lower in order to descend). In the SI system, the mass is in units of Kg. The step accelerates in the -y direction since, according to Newton's law, the acceleration is in the direction of the force. This acceleration is very small because of the large mass of the earth. The step, according to Newton's third law, exerts an equal and opposite force on you (in the positive y direction) and is given by:

$$\vec{F}_{\text{from step}} = -m\vec{g} = +mg\hat{y}\ .$$

The **step** does an amount of work on your body which is given by:

$$W = \vec{F}\bullet(\Delta y) = \left(-(-mg\hat{y})\right)\bullet\left(\vec{y}_{\text{final}} - \vec{y}_{\text{initial}}\right).$$

If $\left(\vec{y}_{\text{final}} - \vec{y}_{\text{initial}}\right) > 0$,in which case you are climbing the stairs, then the work done on your body would be positive, resulting in an increase of your potential energy. If $\left(\vec{y}_{\text{final}} - \vec{y}_{\text{initial}}\right) < 0$, in which case you are descending, the work done on your body would be negative, resulting in a decrease of your potential energy.

In general, note that the work on a body **due to a force from an external agent** is:

Non-calculus	Calculus
$W = \sum_{\text{path}} \vec{F}_i \bullet (\Delta\vec{s}_i)$ i is a segment over which F is constant	$W = \int_{\text{path}} \vec{F}\bullet d\vec{s}$

If the force is conservative, a class of forces including gravitational forces and electrostatic forces, then the work done is independent of the path. If the force is non-conservative, of which friction is an example, then the work done against non-conservative forces depends upon the path. For our purposes today, we will consider that the body is only working against the conservative force of gravity. In this case, the work required reduces to:

Non-calculus	Calculus
$W = \sum_i \vec{F}_i \bullet (\Delta \vec{x}_i)$ i is a segment over which F is constant	$W = \int_{x_{initial}}^{x_{final}} \vec{F} \bullet d\vec{x}$

Both forms become very simple in the additional case of a constant force:

$$W = \vec{F} \bullet (\Delta \vec{s}) .$$

Notice that work is a scalar while force and displacement are both vectors. This is important in understanding that the step may do positive work or negative work on your body, depending upon the sign of the displacement vector, $\Delta \vec{s}$. Also note that work is not a conserved quantity: there is no such thing as ΔW.

By the work-energy theorem, this work results in a change in energy. In the case of walking up stairs, what has changed is potential energy. Thus the change in potential energy would be given by: $\Delta U = W = mg(\Delta y)$.

Work, potential energy (and kinetic energy) have units of Joules (J) in the SI system.
The power exerted by the external agent in a time Δt is the rate of doing work which is:

$$\text{Power} = \tfrac{\Delta U}{\Delta t} = \tfrac{W}{\Delta t} .$$

In the SI system, power has units of Watts (W).

Later in the class, we will discuss impulse, however it is convenient to introduce this now. It is also possible to calculate the impulse delivered. The impulse (SI units: Newton Seconds (N s)) is defined as

$$\vec{J} = \vec{F}_{avg} \Delta t \,(\text{non} - \text{calculus})\, \text{or}\, \vec{J} = \int \vec{F} dt \,(\text{calculus})$$

When your body does work, this comes at the expense of electrostatic energy stored in the form of chemical bonds. While it is easy to calculate the energy expenditures in climbing stairs, it is not so easy to calculate this when descending. In our lab today, we will ignore the descending calculations.

Procedure

Measure the height of a step, then count the number of steps from the basement of the Derby Center to the second floor. Weight yourself. Walk up and down the steps at a constant rate, using a stopwatch to time yourself. Do this for four different trip times and calculate the power expended in Watts for each trip.

The energy which must be expended in walking to the second floor is given by:

$$U = mgh = mg(\#\text{steps} \times \text{height of 1 step})$$

Height of a step (m)	h	
# steps total to 2nd floor	#	
Body mass (kg)	m	
Energy expended in ascent	U=mg(#h)	

Power calculations

Trip number	Time (s) for trip up stairs	Power (U/t) expended (Watts)
1		
2		
3		
4		

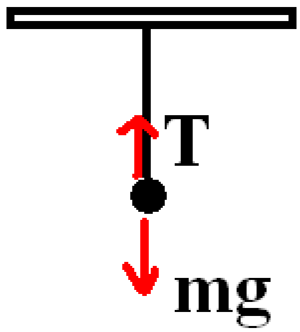
Analysis

Your analysis for this portion of the lab is completed by understanding how to do the appropriate calculations to complete the tables above.

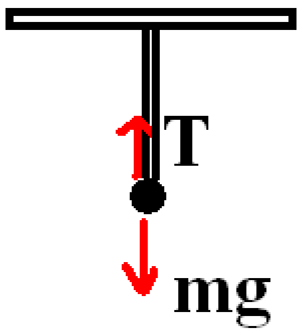
Mechanical Advantage

This is a bit of a tutorial that is designed to help you better understand forces.

Consider the following problem: A point mass (m) is held up by a string. What is the tension in the string?



Clearly, the tension is T=mg.
Now double the string in a funny way: imagine cutting it and attaching it to the mass as shown. What is the tension in each segment now?



Clearly, the total tension in each string must add up to the total weight being suspended. The tension is equally divided between the two segments. Thus, the tension in each string is now T=1/2 mg.

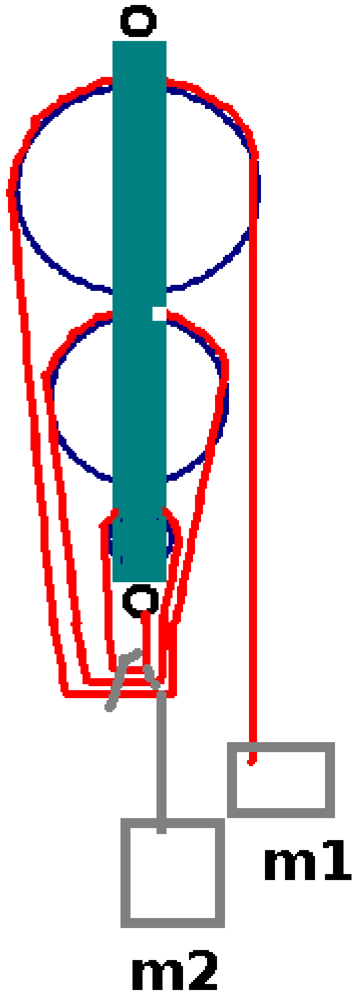
I believe now it is easy to see that if this process is continued you can see that the tension in each segment where there are n total segments would be given by:

$$T = \frac{mg}{n}$$

Now suppose that one of the segments of string were cut and you were required to provide the tension that the upper holder was previously providing. How much force would you need to supply if there were n segments? Again the answer is:

$$T = \frac{mg}{n}$$

This is the basis for understanding mechanical advantage and we'll experiment with it using pulleys to verify this behavior. The ultimate consequence of mechanical advantage is that it takes much less force to hold up an object than the actual weight of the object provided that your rope and pulley system is designed correctly. A correct design in a world filled with friction pretty much requires that each rope segment be wound over a pulley. We'll modify this slightly.



Procedure:

Wind the thread through the three pulleys as shown. Place about 200 to 250 g on hanger 2. Put mass on hanger 1 until the system does not move. This is not actually the point at which the two masses are correctly determined, however, because of friction in the pulleys. You need to place additional mass on hanger 1 until about the same amount of force is required to pull the system one direction as the other. Record these masses.

Now hold a meter stick up to the system and align mass m2 to a dot on the string for mass m1. You should use a sharpie to make the dot on the string. Raise mass m2 through a distance of 5 cm and find out how much below the original position the dot on the string has moved.

Now rethread your system, leaving out the large pulley and repeat.

Repeat one more time for only the smallest pulley in the system.

I have constructed a spreadsheet to help with today's calculations. First you should note the ratio between the force from mass 1 and the force from mass 2. From your data, you should be able to determine if the tension in any one string is the total weight suspended divided by the number of strings or not.

Now for additional calculations:

When mass m1 is moved through a distance Δx, mass m2 is moved through a distance also. If you observe that these two distances are the same, you need to observe again.
However, there is a quantity which is the same in both cases, namely the work done. Work is defined as a force exerted through a distance. Here, this force results in m1 being lowered while m2 is raised. In pulling m1 down, the displacement is in the same direction as the force while as at the same time m2 moves in the opposite direction of the force. Overall, however, if "work in = work out" which, in this situation also equates to conservation of energy, you should find that the following is true (to within experimental error):

$$m_2 g\Delta x_2 + m1 g\Delta x_1 = 0$$

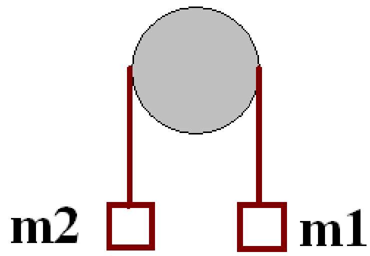
This calculation is done on your worksheet for you with the % error determined by:

$$\%\text{error} = \frac{m_1 g\Delta x_1 + m_2 g\Delta x_2}{\frac{1}{2}[m_1 g\Delta x_1 + m_2 g\Delta x_2]} \times 100$$

From your work, you should be able to determine if the overall change in potential energy is zero for this system (to within experimental error).

Atwood's machine

In class, we have seen how Atwood's machine works, and have done the analysis on it. If you will recall, Atwood's machine consists of a single pulley with two masses.



Now from the analysis in class assuming m1 is greater than m2, you can determine the acceleration of this system which is given by:

$$a = \frac{m_1 - m_2}{m_1 + m_2} g$$

In previous versions of this lab, Atwood's machine was used to measure the value of g but errors associated with friction, the inertial of the pulley, and timing prevent very good measurements from resulting. We will later be able to measure g with the simple pendulum. Today, however, I want you to construct Atwood's machine and observe several situations only. In particular, I want you to observe what happens when the two masses are equal, and I also want you to observe what happens when m1 is larger than m2 and also what happens when m2 is larger than m1. In particular, you should note the amount of time required for the mass to move through a specified distance and relate this to the acceleration. Your analysis on this part is permitted to be in the form of a narrative detailing the analysis (from class) of the Atwood's machine and also your observations.