	Physics 210: Worksheet 15	Name ________________

Each of these problems involve physics in an accelerated frame of reference
Although your mind wants to try to force you to work these problems inside the accelerated reference frame (i.e. the so-called "wrong way" by some people), you should force your mind to do these the right way by direct application of Newton's laws and no introduction of fictional forces!

In each of these problems, the moral to the story is do what ever you can to make sure you stay outside the accelerated reference frame and FORCE your mind to stay there!

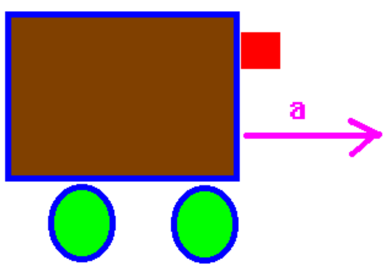
(1) Suppose a roller coaster ride has a loop of radius r. What is the minimum speed that a roller coaster can have and still stay on the track at the top?

(2) Suppose a hollow spherical space station of radius R is in space and is rotating about one axis. With what angular velocity must the space station rotate so that a person located at the equator of the space station experiences an acceleration equal to g? How does this acceleration vary as the person moves towards the pole along the sphere?

(3) How long would the day be on the Earth if an object at the Earth's equator would float? How much is the apparent weight of a 100 kg object changed at the equator due to the rotation of the Earth?

(4) A train car has a mass hanging by some fishing line from its roof. How much acceleration must the train car experience in order to produce an angle of $\phi=25^0$ between the string and the vertical. What happens if the train is moving with a constant velocity?

(5) A cart is undergoing an acceleration as shown. A block is on the side of the cart. If there is a static coefficient of friction μ between the block and the cart, how much must the acceleration be so that the block does not slide?



(6) Suppose you find yourself in a giant centrifuge of radius R. The walls of the centrifuge have a static coefficient of friction of μ with your back. What must be the frequency (f) with which the centrifuge rotates so that you will remain stuck to the wall?

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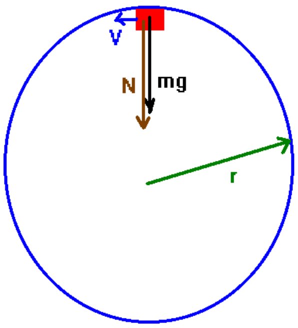
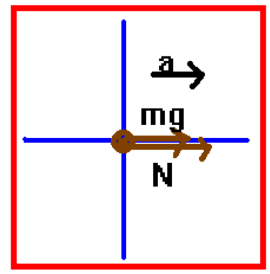
(1) Suppose a roller coaster ride has a loop of radius r. What is the minimum speed that a roller coaster of mass m can have and still stay on the track at the top?



Solution:

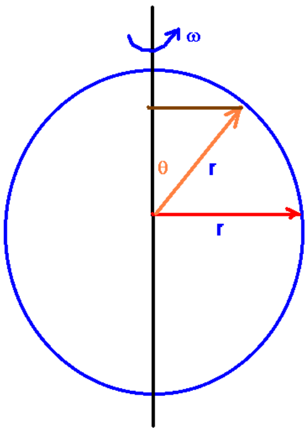
There are 2 ways to look at this problem ... the right way and the wrong way. We want to concentrate on doing things the right way so I won't show the wrong way here.

The track is exerting a normal force on the cart. Let's look at a free body diagram of the system. *I recommend that if possible you make a mental snapshot of things and rotate to produce a positive acceleration when ever possible.*

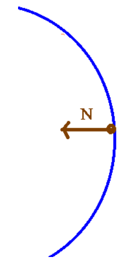


According to Newton's laws: $\sum \vec{F} = m\vec{a}$ and so here, we have $N + mg = ma_c$. The cart will fall when the normal force is equal to zero. Thus, this requires $a_c = g$. But the centripetal acceleration is given by $a_c = \frac{v^2}{r}$ so we can then say that the desired relation looks like $g = \frac{v^2}{r} \Rightarrow v = \pm\sqrt{rg}$. I encourage you not to transfer yourself to the frame of reference of the cart in this problem since that introduces forces which are really not present. Sometimes, it's really hard to force your mind into thinking about things like this. Also please note, this problem is not really uniform circular motion although it is indeed closely related (the speed is lower on the top than it is on the bottom).

(2) Suppose a hollow spherical space station of radius R is in space and is rotating about one axis. With what angular velocity must the space station rotate so that a person located at the equator of the space station experiences an acceleration equal to g? How does this acceleration vary as the person moves towards the pole along the sphere?



Here is a picture of the situation. Along the equator, we have the following situation



The only force present here is the Normal force and that is the force which is providing the centripetal acceleration. We thus apply Newton's laws to this situation:

$\sum \vec{F} = ma$. We want to simulate gravity here so we ultimately want this force to be equal to mg. Thus, we require $ma = m\frac{v^2}{r} = mg \Rightarrow v = \pm\sqrt{rg}$. But, the angular velocity is related to v by $v = \omega r$. So it is clear that we have the requirement:

$$\pm\sqrt{rg} = \omega r \Rightarrow rg = \omega^2 r^2 \Rightarrow \omega = \pm\sqrt{\frac{g}{r}}.$$

The problem of what happens away from the equator is just a bit more complicated than this, however. Let x represent the perpendicular distance from this axis. This distance is related to r by $x = r\sin(\theta)$. The tangential velocity, however, is going to change. If we keep the same ω as in the first part, we can find the tangential velocity to be given by:

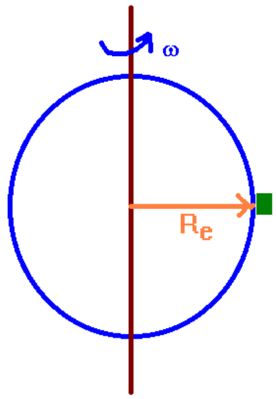
$$v = \omega x = \sqrt{\tfrac{g}{r}}\, r\sin(\theta).$$

The centripetal acceleration at each point is then given by

$$a_c = \frac{v^2}{x} = \frac{\left(\sqrt{\frac{g}{r}}\, r\sin(\theta)\right)^2}{r\sin(\theta)} = \frac{rg\sin^2(\theta)}{r\sin(\theta)} = g\sin(\theta)$$

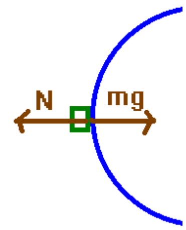
which correctly predicts $a_c \to 0$ at the poles, and a_c=g at the equator.

(3) How long would the day be on the Earth if an object at the Earth's equator would float? How much is the apparent weight of a 100 kg object changed at the equator due to the rotation of the Earth?
Solution: The situation is shown below:



So long as the normal force is less than the weight of the object, the acceleration will be centripetal and directed towards the center of the Earth as shown in the second figure.

Let's apply Newton's laws to this situation: $\sum \vec{F} = m\vec{a}$. Application of Newton's laws then says $mg - N = ma_c$. The object will float when N=0. Thus we have $mg = ma_c$, or ultimately $a_c = g$.



Now since

$$a_c = \frac{v^2}{R_e} = \frac{\omega^2 R_e^2}{R_e} = \omega^2 R_e$$

we have the result given as:

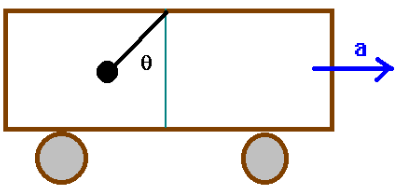
$$R_e\omega^2 = g \Rightarrow \omega = \sqrt{\frac{g}{R_e}} = 2\pi f \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{R_e}} \Rightarrow T = 2\pi\sqrt{\frac{R_e}{g}}$$

Here f is frequency and T is period.

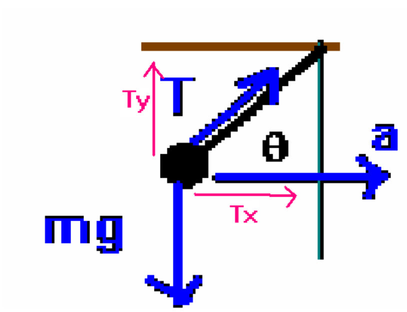
We can carry this a little bit further to find out what how much the weight is different from mg at the equator. Probably it's best to imagine a scale under a person at the equator. The scale is reading the normal force which it is exerting. Thus, we have $mg - mR_e\omega^2 = N \Rightarrow m(g - R_e\omega^2) = N$ We can thus reach the following answers: using $R_e = 6.37\text{x}10^6$ we find that $T = 2\pi\sqrt{\frac{6.37\text{x}10^6}{9.8}} = 5066\text{s} = 84\,\text{min} = 1.4\text{hr}$. Now, the angular frequency of the Earth is $\omega = 2\pi f = \frac{2\pi}{T} = \frac{2\pi}{86400} = 7.27\text{x}10^{-5}\ \frac{\text{"rad"}}{\text{s}}$. So to find out the apparent weight of an object, we have $N = m(g - R_e\omega^2) = 100(9.8 - 6.37\text{x}10^6(7.27\text{x}10^{-5})^2) = 977\text{N}$ This is compared to 980N for a 100kg object at the north pole if the Earth were perfectly spherical (i.e. it's 3 N less at the equator).

(4) A train car has a mass hanging by some fishing line from its roof. How much acceleration must the train car experience in order to produce an angle of $\phi=25^0$ between the string and the vertical. What happens if the train is moving with a constant velocity?

This is your first problem in which you must resolve components of a force ...it is not difficult to do.

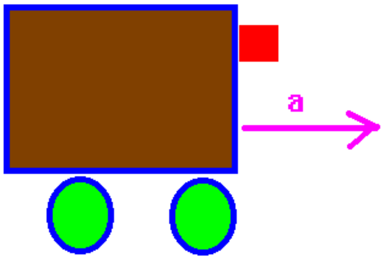


Solution: The situation is shown to the left. In this case, it is the tension in the string which is producing the acceleration of the mass. Let's draw in each of the forces and analyze this problem not from the accelerated reference frame (which some would call the "wrong" way) but rather from outside this frame.

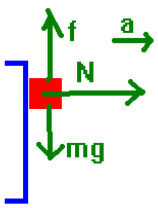


We have the forces which I have shown here. Apply Newton's laws to this situation: $\sum \vec{F} = m\vec{a}$. Along the y direction, we then have $T_y - mg = 0$ Along the x direction we have: $T_x = ma$. Now we have a connection between T_x, T_y and the angle which is given by $\frac{T_y}{T_x} = \tan(\theta)$. We can find the value of T_y by looking at the y-equation of motion: $T_y = T\sin(\theta) = mg \Rightarrow T = \frac{mg}{\sin(\theta)}$ which is the tension in the string. We can then find the required acceleration from the x equation: $T_x = T\cos(\theta) = \frac{mg}{\sin(\theta)}\cos(\theta) = mg\cot(\theta) = \frac{mg}{\tan(\theta)} = ma$. This then gives the required acceleration as $a = g\cot(\theta)$ If $\phi=25^0$ then $\theta=90-25=65^0$. Thus, $a = 9.8\cot(65) = 4.57 m/s^2$. You can verify in terms of ϕ that the required acceleration is $a = g\tan(\phi)$.

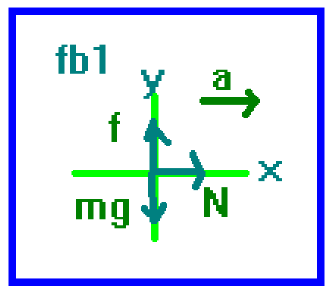
(5) A cart is undergoing an acceleration as shown. A block is on the side of the cart. If there is a static coefficient of friction μ between the block and the cart, how much must the acceleration be so that the block does not slide?



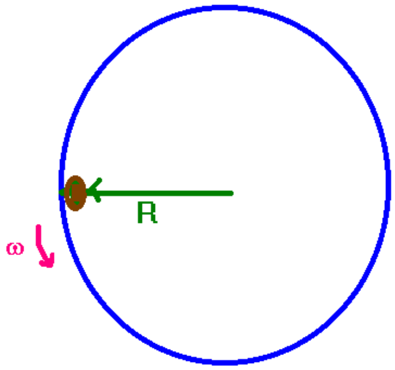
Solution: You need to analyze this problem looking at the forces involved. Clearly, if there were no frictional force, the block would fall off. Also, if there were no normal force, the block would not accelerate. If there were no weight, the block would not fall. If there were no acceleration, the problem wouldn't work ... etc. Let's sketch the free body diagram for this situation.



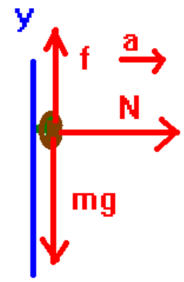
A sketch of the forces involved looks like this:
The free body diagram is shown to the right.
We apply Newton's laws to this situation:
$\sum \vec{F} = m\vec{a}$. This gives us the following equations of motion:
$f - mg = 0$ and $N = ma$. The frictional force here is clearly in the direction shown. You can set the coefficient to zero to see which way it slides ... friction is in the opposite direction. The frictional force is still given by $f = \mu N$ but you must resist the temptation (by the evil one perhaps) to say that this is mg. In fact, here the normal force is shown above and is equal to ma. We then have $\mu ma - mg = 0 \Rightarrow a = \frac{g}{\mu}$.



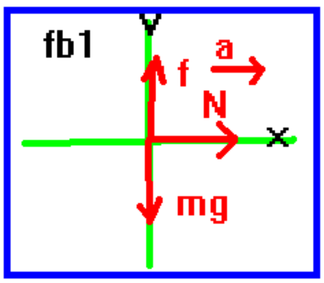
(6) Suppose you find yourself in a giant centrifuge of radius R. The walls of the centrifuge have a static coefficient of friction of μ with your back. What must be the frequency (f) with which the centrifuge rotates so that you will remain stuck to the wall?



This situation is shown to the left. You need to analyze this problem only in terms of the forces which are actually there. As usual, you want to draw a free body diagram for this system. A sketch of the situation (where I have rotated the system to insure better visualization of the forces) is shown to the right.



The free body diagram for this system looks like what is shown below.



We apply Newton's laws to this system: $\sum \vec{F} = m\vec{a}$. The frictional force is still given by f=μN and again you've got to resist that temptation to say this is mg. Let's write out the components of Newton's laws:

$N = ma$ for the x-direction and $f - mg = 0$ for the y direction. We can solve for the frictional force now:

$$f = \mu N = \mu m a_c$$

since the acceleration present is, for sure, the centripetal acceleration. Use this in the y-equation of motion to obtain:

$$\mu m a_c - mg = 0 \Rightarrow \mu a_c = g \Rightarrow a_c = \frac{g}{\mu} = \frac{v^2}{R} = \frac{\omega^2 R^2}{R} = \omega^2 R.$$

Let's solve this for ω:

$$\omega^2 R = \frac{g}{\mu} \Rightarrow \omega^2 = \frac{g}{\mu R} \Rightarrow \omega = \sqrt{\frac{g}{\mu R}} = 2\pi f \Rightarrow f = \frac{1}{2\pi}\sqrt{\frac{g}{\mu R}}$$